

Time: 1 Hour

Total Marks:20

- N.B.: (1) Attempt any four questions
 (2) Figures to the right indicate full marks

Q1. Prove that $\left(\frac{1 + \tanh x}{1 - \tanh x}\right)^3 = \cosh 6x + \sinh 6x$ [05]

Q2. Show that $x^5 - 1 = (x - 1)\left(x^2 + 2x \cos \frac{\pi}{5} + 1\right)\left(x^2 + 2x \cos \frac{3\pi}{5} + 1\right)$ [05]

Q3. Using DeMoivre's Theorem prove that $2(1 + \cos 8\theta) = (x^4 - 4x^2 + 2)^2$ where $x = 2 \cos \theta$ [05]

Q4. Simplify $\left(\frac{1 + \sin \frac{\pi}{8} + i \cos \frac{\pi}{8}}{1 + \sin \frac{\pi}{8} - i \cos \frac{\pi}{8}}\right)^8$ [05]

Q5. Prove that $\sin^{-1}(\operatorname{cosec} \theta) = \frac{\pi}{2} + i \log \cot \frac{\theta}{2}$ [05]

Q6. If $\tan\left(\frac{\pi}{3} + i\alpha\right) = x + iy$ prove that $x^2 + y^2 - \frac{2x}{\sqrt{3}} = 1$ [05]

Q7. If $\frac{(1+i)^x + iy}{(1-i)^x - iy} = \alpha + i\beta$ then considering only principle values, prove that [05]

$$\tan^{-1}\left(\frac{\beta}{\alpha}\right) = \frac{\pi x}{2} + y \log 2$$

$\sin(\theta) = \cos(\theta)$
 $\theta = \dots$

Q8. If $\sin^4 \theta \cos^3 \theta = a_1 \cos \theta + a_3 \cos 3\theta + a_5 \cos 5\theta + a_7 \cos 7\theta$ prove that [05]

$$a_1 + 9a_3 + 25a_5 + 49a_7 = 0$$

$\frac{2 \times 180}{3}$