

ATTEMPT ANY FOUR :-

(1) Show that $(4n)$ th power of $\frac{1+7i}{(2-i)^2}$ is equal to $(-4)^n$
Where n is a positive integer. (5)

(2) Show that

$$\tan 7\theta = \frac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}$$
 (5)

(3) Using De Moivre's Theorem prove that

$$\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$$
 (5)

(4) If $u = \log_e \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, Prove that $\tanh u = \tan \frac{\theta}{2}$
and $\cosh u \cos \theta = 1$ (5)

(5) Prove that

$$\log\left(\frac{1}{1-e^{i\theta}}\right) = \log\left(\frac{1}{2} \operatorname{cosec} \frac{\theta}{2}\right) + i\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$
 (5)