

Unit Test-I

Time 1hr

Mark-20

Q.1 Attempt any five

(10)

- a) Solve  $x^6 - i = 0$
- b) Prove that  $\cosh^{-1} \sqrt{1+x^2} = \sinh^{-1} x$
- c) ST  $\log(-\log i) = \log \frac{\pi}{2} - i \frac{\pi}{2}$
- d) P.T  $(1+i)^{10} + (1-i)^{10} = 0$
- e) Define Hermitian matrix and skew Hermitian matrix
- f) Show that matrix  $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$  is unitary if  $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$
- g) If A is orthogonal matrix then prove that  $|A| \pm 1$
- h) Prove the matrix A is orthogonal  $\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Q.2 Express following matrix as sum of Hermitian and skew Hermitian matrix (5)

$$A = \begin{bmatrix} 2 & 3-i & 2+i \\ i & 0 & 1-i \\ 1+2i & 1 & 3i \end{bmatrix}$$

Or

Prove that every square matrix can be uniquely expressed as  $P + iQ$  where P and Q are Hermitian matrices

Q.3 If  $\sinh(\theta + i\varphi) = e^{i\alpha}$  then prove that  $\sinh^4 \theta = \cos^2 \alpha = \cos^4 \varphi$  (5)

Or

If  $\sin 6\theta = a \cos^5 \theta \sin \theta + b \cos^3 \theta \sin^3 \theta + c \cos \theta \sin^5 \theta$  find value of a, b, c.