

## Solution to IIT JEE 2019 (Advanced) : Paper - I

### PART I – PHYSICS

#### PAPER –1 : INSTRUCTIONS TO CANDIDATES

- Question Paper–1 has three (03) parts : Physics, Chemistry and Mathematics.
- Each part has a total eighteen (18) questions divided into three (03) sections (Section–1, Section–2 and Section–3)
- Total number of questions in Question Paper-1 are Fifty Four (54) and Maximum Marks are One Hundred Eighty Six (186)

#### Type of Questions and Marking Scheme

##### SECTION 1 (Maximum Marks:12)

- This section contains FOUR (04) questions.
- Each question has FOUR options ONLY ONE of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :  
Full Marks : +3 If ONLY the correct option is chosen.  
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).  
Negative Marks : –1 In all other cases.

##### SECTION 2 (Maximum Marks : 32)

- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the correct option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :  
Full Marks : +4 If only (all) the correct option(s) is (are) chosen.  
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.  
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.  
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.  
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).  
Negative Marks : –1 in all other cases

##### SECTION 3 (Maximum Marks:18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to TWO decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :  
Full Marks : +3 If ONLY the correct numerical value is entered.  
Zero Marks : 0 In all other cases.

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### **Answering Questions :**

- To select the option(s), use the mouse to click on the corresponding button(s) of the option(s).
- To deselect the chosen option for the questions of SECTION–1, click on the button of the chosen option again or click on the Clear Response button to clear the chosen option.
- To deselect the chosen option(s) for the questions of SECTION–2, click on the button(s) of the chosen option(s) again or click on the Clear Response button to clear all the chosen options.
- To change the option(s) of a previously answered question of SECTION–1 and SECTION–2 first deselect as given above and then select the new option(s).
- To answer questions of SECTION–3, use the mouse to click on numbers (and/or symbols) on the on–screen virtual numeric keypad to enter the numerical value in the space provided for answer.
- To change the answer of a question of SECTION–3, first click on the Clear Response button to clear the entered answer and then enter the new numerical value.
- To mark a question ONLY for review (i.e. without answering it), click on the Mark for Review & Next button.
- To mark a question for review (after answering it), click on Mark for Review & Next button – the answered question which is also marked for review will be evaluated.
- To save the answer, click on the Save & Next button – the answered question will be evaluated.

**PART I – PHYSICS**

**SECTION 1 (Maximum Marks:12)**

- This section contains FOUR (04) questions.
- Each question has FOUR options ONLY ONE of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :  
 Full Marks : +3 If ONLY the correct option is chosen.  
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).  
 Negative Marks : -1 In all other cases.

1. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is  $V_0$ . A hole with small area  $\alpha 4\pi R^2$  ( $\alpha \ll 1$ ) is made on the shell without affecting the rest of the shell. Which one of the following statement is correct.

- (A) The ratio of the potential at the center of the shell to that of the point at  $\frac{1}{2}R$  from center towards the hole will be  $\frac{1-\alpha}{1-2\alpha}$
- (B) The potential at the centre of shell is reduced by  $2\alpha V_0$
- (C) The magnitude of electric field at the center of the shell is reduced by  $\frac{\alpha V_0}{2R}$
- (D) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance  $2R$  from the center of the spherical shell will be reduced by  $\frac{\alpha V_0}{2R}$

1. (A)

Potential at surface,  $V_0 = \frac{KQ}{R}$

Potential at C

$$V_C = \frac{KQ}{R} - \frac{K\alpha Q}{R} = V_0(1 - \alpha)$$

Potential at B

$$V_B = \frac{KQ}{R} - \frac{K(\alpha Q)}{R/2} = V_0(1 - 2\alpha)$$

$$\therefore \frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}$$

Electric field at A

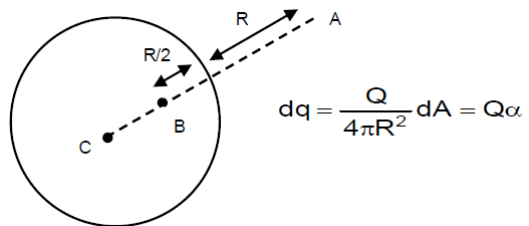
$$E_A = \frac{KQ}{(2R)^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$$

So reduced by  $\frac{\alpha V_0}{R}$

Electric field at C

$$E_C = \frac{K(\alpha Q)}{R^2} = \frac{\alpha V_0}{R}$$

So increased by  $\frac{\alpha V_0}{R}$



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2. In a radioactive sample  ${}^{40}_{19}\text{Ar}$  nuclei either decay into stable  ${}^{40}_{20}\text{Ca}$  nuclei with decay constant  $4.5 \times 10^{-10}$  per year or into stable  ${}^{40}_{18}\text{Ar}$  nuclei with decay constant  $0.5 \times 10^{-10}$  per year. Given that in this sample all the stable  ${}^{40}_{20}\text{Ca}$  and  ${}^{40}_{18}\text{Ar}$  nuclei are produced by the  ${}^{40}_{19}\text{K}$  nuclei only. In time  $t \times 10^9$  years, if the ratio of the sum of stable  ${}^{40}_{20}\text{Ca}$  and  ${}^{40}_{18}\text{Ar}$  nuclei to the radioactive  ${}^{40}_{19}\text{Ar}$  nuclei is 99, the value of  $t$  will be [Given  $\ln 10 = 2.3$ ]  
 (A) 9.2 (B) 1.15 (C) 4.6 (D) 2.3

2. (A) Parallel radioactive decay. So equivalent decay constant

$$\lambda = \lambda_1 + \lambda_2 = 5 \times 10^{-10} \text{ per year}$$

$$N = N_0 e^{-\lambda t}$$

$$N_0 - N = N_{\text{stable}}$$

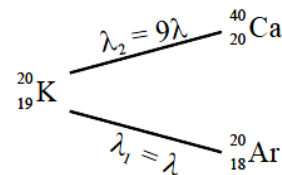
$$N = N_{\text{radioactive}}$$

$$\frac{N_0}{N} - 1 = 99$$

$$\frac{N_0}{N} = 100$$

$$\frac{N}{N_0} = e^{-\lambda t} = \frac{1}{100} \Rightarrow \lambda t = 2 \ln 10 = 4.6$$

$$t = 9.2 \times 10^9 \text{ years}$$



3. A current carrying wire heats a metal rod. The wire provides a constant power ( $P$ ) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature ( $T$ ) in the metal rod changes with time ( $t$ ) as

$$T(t) = T_0(1 + \beta t^{1/4})$$

where  $\beta$  is a constant with appropriate dimension while  $T_0$  is a constant with dimension of temperature. The heat capacity of metal is :

(A)  $\frac{4P(T(t) - T_0)^4}{\beta^4 T_0^5}$

(B)  $\frac{4P(T(t) - T_0)^3}{\beta^4 T_0^4}$

(C)  $\frac{4P(T(t) - T_0)}{\beta^4 T_0^2}$

(D)  $\frac{4P(T(t) - T_0)^2}{\beta^4 T_0^3}$

3. (B)

Given,  $P = \frac{dQ}{dt}$  and  $T(t) = T_0(1 + \beta t^{1/4})$

At equilibrium,  $C \frac{dT}{dt} = P$

$$\frac{dT}{dt} = \frac{T_0 \beta}{4} t^{-3/4}$$

So heat capacity  $C = \frac{4P}{\beta T_0} t^{3/4}$

From the given equation  $\frac{T(t) - T_0}{\beta T_0} = t^{1/4}$

So,  $t^{3/4} = \frac{(T(t) - T_0)^3}{\beta^3 T_0^3}$ . So,  $C = \frac{4P}{\beta^4 T_0^4} (T(t) - T_0)^3$

4. Consider a spherical gaseous cloud of mass density  $\rho(r)$  in a free space where  $r$  is the radial distance from its centre. The gaseous cloud is made of particles of equal mass  $m$  moving in circular orbits about their common centre with the same kinetic energy  $K$ . The force acting on the particles is their mutual gravitational force. If  $\rho(r)$  is constant in time. The particle number density  $n(r) = \rho(r)/m$  : ( $G =$  universal gravitational constant)

(A)  $\frac{K}{6\pi r^2 m^2 G}$       (B)  $\frac{K}{\pi r^2 m^2 G}$       (C)  $\frac{3K}{\pi r^2 m^2 G}$       (D)  $\frac{K}{2\pi r^2 m^2 G}$

4. (D)

Let total mass included in a sphere of radius  $r$  be  $M$ .

For a particle of mass  $m$ ,

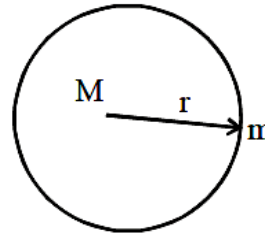
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GMm}{r} = 2K \quad \Rightarrow M = \frac{2Kr}{Gm}$$

$$\therefore dM = \frac{2Kdr}{Gm}$$

$$\Rightarrow (4\pi r^2 dr)\rho = \frac{2Kdr}{Gm} \quad \Rightarrow \rho = \frac{K}{2\pi r^2 Gm}$$

$$\therefore n = \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G}$$



**SECTION 2 (Maximum Marks:32)**

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the correct option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :
 

Full Marks	: +4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: +3	If all the four options are correct but <b>ONLY</b> three options are chosen.
Partial Marks	: +2	If three or more options are correct but <b>ONLY</b> two options are chosen and both of which are correct.
Partial Marks	: +1	If two or more options are correct but <b>ONLY</b> one option is chosen and it is a correct option.
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	: -1	in all other cases

1. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of  $L$ , which of the following in statement(s) is/are correct?

- (A) The dimension of force is  $L^{-3}$   
 (B) The dimension of power is  $L^{-5}$   
 (C) The dimension of energy is  $L^{-2}$   
 (D) The dimension of linear momentum is  $L^{-1}$

1. (A), (C), (D)

$$[M^0 L^0 T^0] = [ML^2 T^{-1}] \Rightarrow [L^2] = [T]$$

$$[\text{Energy}] = [MLT^{-2}L] = [L^{-2}]$$

$$[\text{Force}] = [MLT^{-2}] = [L^{-3}]$$

$$[\text{Power}] = [MLT^{-2}LT^{-1}] = [L^{-4}]$$

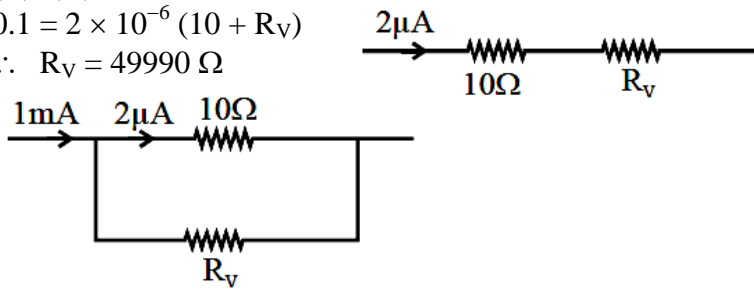
$$[\text{Linear momentum}] = [MLT^{-1}] = [L^{-1}]$$

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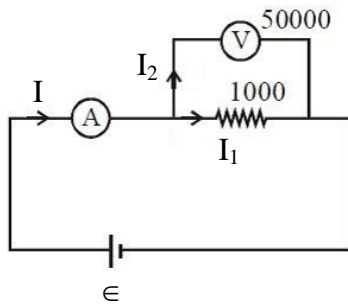
2. Two identical moving coil galvanometers have  $10\ \Omega$  resistance and full scale deflection at  $2\ \mu\text{A}$  current. One of them is converted into a voltmeter of  $100\ \text{mV}$  full scale reading and the other into a Ammeter of  $1\ \text{mA}$  full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with  $R=1000\ \Omega$  resistor by using an ideal cell. Which of the following statement(s) is/are correct ?
- (A) The resistance of the Voltmeter will be  $100\ \text{k}\ \Omega$   
 (B) The resistance of the Ammeter will be  $0.02\ \Omega$  (round off to 2nd decimal place)  
 (C) If the ideal cell is replaced by a cell having internal resistance of  $5\ \Omega$  then the measured value of  $R$  will be more than  $1000\ \Omega$   
 (D) The measured value of  $R$  will be  $978\ \Omega < R < 982\ \Omega$
2. (B), (D)

$$0.1 = 2 \times 10^{-6} (10 + R_V)$$

$$\therefore R_V = 49990\ \Omega$$



$$2 \times 10^{-6} \times 10 = 10^{-3} R_A \quad \therefore R_A = 0.02\ \Omega$$



$$\text{So, } I_2 \cdot 50000 = (I - I_2) \cdot 1000$$

$$\therefore 51I_2 = I$$

$$\therefore \text{Reading} = \frac{I_2 \cdot 50000}{I} = 980$$

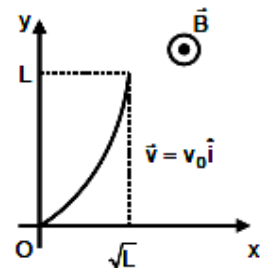
$$I = \frac{51\epsilon}{5 \times 10^4} \quad (\because R_A \rightarrow 0)$$

$$I_1 = \frac{\epsilon}{1000}$$

3. A conducting wire of parabolic shape,  $y = x^2$ , is moving with velocity  $\vec{V} = V_0 \hat{i}$  in a non-uniform magnetic field

$$\vec{B} = B_0 \left[ 1 + \left( \frac{y}{L} \right)^\beta \right] \hat{k}, \text{ as shown in figure. If } V_0, B_0, L \text{ and } \beta$$

are positive constants and  $\Delta\phi$  is the potential difference developed between the ends of the wire, then the correct statements(s) is/are :



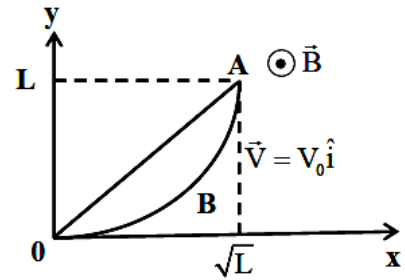
- (A)  $|\Delta\phi| = \frac{4}{3} B_0 V_0 L$  for  $\beta = 2$   
 (B)  $|\Delta\phi|$  remains same if the parabolic wire is replaced by a straight wire,  $y = x$ , initially, of length  $\sqrt{2}L$   
 (C)  $|\Delta\phi| = \frac{1}{2} B_0 V_0 L$  for  $\beta = 0$   
 (D)  $|\Delta\phi|$  is proportional to the length of wire projected on  $y$ -axis

3. (A), (B), (D)

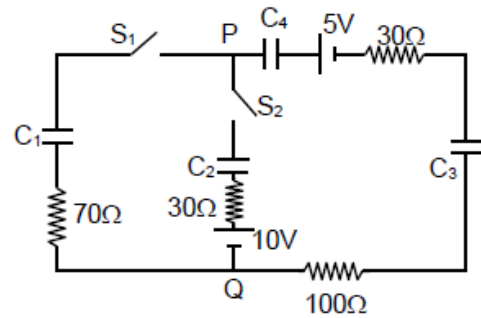
There is no change in flux through the loop OABO due to the movement of loop. So potential difference developed in curved wire and the straight wire OA is same.

For  $\beta = 0$ ,  $|\Delta\phi| = 2B_0V_0L$

For  $\beta = 2$ ,  $|\Delta\phi| = \int_0^L B_0 \left(1 + \frac{y^2}{L^2}\right) V_0 dy = \frac{4}{3} B_0 V_0 L$

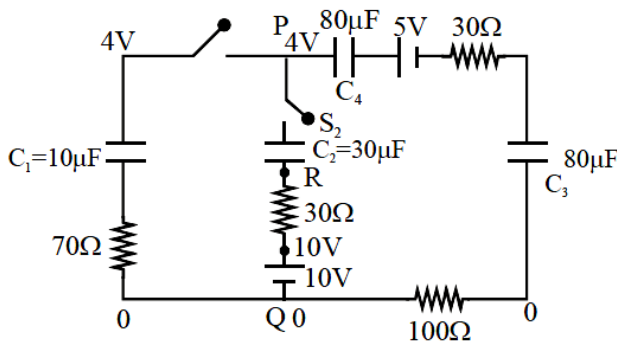


4. In the circuit shown, initially there is no charge on capacitors and keys  $S_1$  and  $S_2$  are open. The values of the capacitors are  $C_1 = 10\mu\text{F}$ ,  $C_2 = 30\mu\text{F}$ , and  $C_3 = C_4 = 80\mu\text{F}$ . Which statements is/are correct :

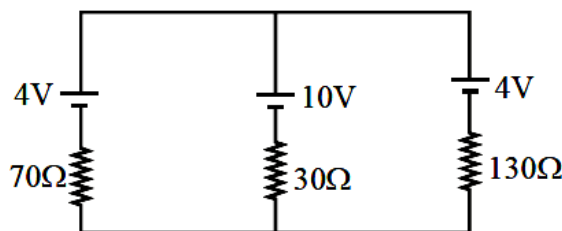


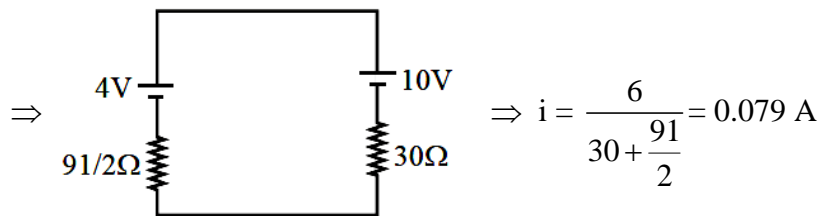
- (A) The key  $S_1$  is kept closed for long time such that capacitors are fully charged. Now key  $S_2$  is closed, at this time the instantaneous current across  $30\Omega$  resistor (between points P & Q) will be 0.2A round off to 1st decimal place).
- (B) If key  $S_1$  is kept closed for long time such that capacitors are fully charged, the voltage across  $C_1$  will be 4V.
- (C) At time  $t = 0$ , the key  $S_1$  is closed, the instantaneous current in the closed circuit will be 25 mA
- (D) If  $S_1$  is kept closed for long time such that capacitors are fully charged, the voltage difference between P and Q will be 10V.

4. (B), (C)



When  $S_2$  is closed at  $t = 0^+$ , capacitor  $C_1$  acts as a battery of 4V,  $C_4$  and  $C_3$  of  $\frac{1}{2}$  V each,  $C_2$  is shorted circuit will look like





At steady state,

When capacitor is fully charged it behave as open circuit and current through it zero.

Hence, charge on each capacitor is same.

$$Q = C_{eq} V = (8 \mu\text{F}) \times 5$$

$$Q = 40 \mu\text{C}$$

$$\text{Now, } V_p - \frac{40}{10} = V_Q$$

$$V_p - V_Q = 4\text{V}$$

At  $t = 0$ , when  $S_1$  is closed, capacitor act as short circuit.

$$i = \frac{V}{R_{eq}} = \frac{5}{200} = 25 \text{ mA}$$

5. A charged shell of radius  $R$  carries a total charge  $Q$ . Given  $\phi$  as the flux of electric field through a closed cylindrical surface of height  $h$ , radius  $r$  and with its center same as that of the shell. Here center of cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is are correct [  $\epsilon_0$  is the permittivity of free space ]

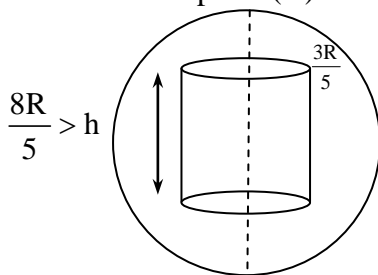
(A) If  $h > 2R$  and  $r = \frac{4R}{5}$  then  $\phi = \frac{Q}{5\epsilon_0}$

(B) If  $h > 2R$  and  $r = \frac{3R}{5}$  then  $\phi = \frac{Q}{5\epsilon_0}$

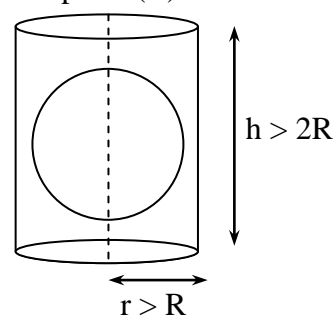
(C) If  $h < \frac{8R}{5}$  and  $r = \frac{3R}{5}$  then  $\phi = 0$

(D) If  $h > 2R$  and  $r > R$  then  $\phi = \frac{Q}{\epsilon_0}$

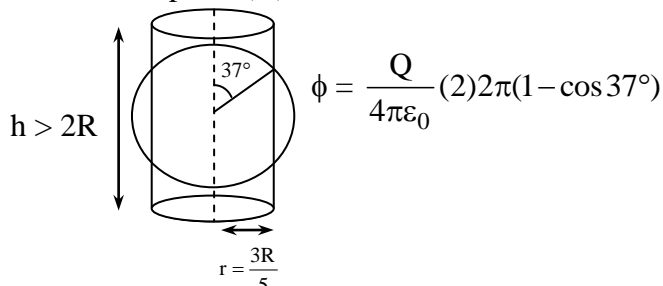
5. (B), (C), (D) Option (A)



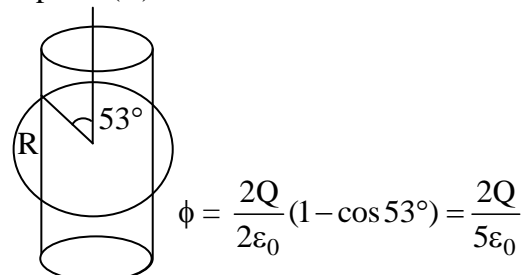
Option (B)



Option (D)

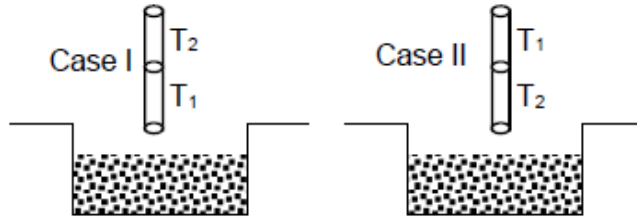


Option (C)





6. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries  $T_1$  and  $T_2$  of different materials having water contact angles of  $0^\circ$  and  $60^\circ$  respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?  
 [Surface tension of water =  $0.075\text{N/m}$ , density of water =  $1000\text{ kg/m}^3$ , take  $g = 10\text{ m/s}^2$ ]



- (A) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)  
 (B) For case I, capillary joint is 5cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)  
 (C) The correction in the height of water column raised in the tube, due weight of water contained in the meniscus, will be different for both cases.  
 (D) For case II, the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)
6. (A), (C), (D)

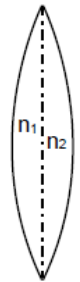
When  $T_1$  is in contact with water, then  $h = \frac{2T \cos \theta_1}{r\rho g} = 7.5\text{ cm} < 8\text{ cm}$

But in option (B) height is insufficient.

When  $T_2$  is in contact with water, then  $h = \frac{2T \cos \theta_2}{r\rho g} = 3.75\text{ cm} < 5\text{ cm}$

Since Angle of contact is different so correction in height will be different in both cases. Volume of water in the meniscus depends upon the angle of contact.'

7. A thin convex lens is made of two materials with refractive indices  $n_1$  and  $n_2$ , as shown in figure. The radius of curvature of the left and right spherical surfaces are equal  $f$  is the focal length of the lens when  $n_1 = n_2 = n$ . The focal length is  $f + \Delta f$  when  $n_1 = n$  and  $n_2 = n + \Delta n$ . Assuming  $\Delta n \ll (n - 1)$  and  $(1 < n < 2)$ , the correct statement(s) is/are



(A) If  $\frac{\Delta n}{n} < 0$  then  $\frac{\Delta f}{f} > 0$

- (B) For  $n = 1.5$ ,  $\Delta n = 10^{-3}$  and  $f = 20\text{ cm}$ , the value of  $|\Delta f|$  will be 0.02 cm (round off to 2nd decimal place).

(C)  $\left| \frac{\Delta f}{f} \right| < \left| \frac{\Delta n}{n} \right|$

- (D) The relation between  $\frac{\Delta f}{f}$  and  $\frac{\Delta n}{n}$  remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

7. (A), (B), (D)

When  $n_1 = n_2 = n$

$$\frac{1}{f} = (n-1) \times \frac{2}{R}$$

So,  $f = \frac{R}{2(n-1)} \dots (1)$



Second case :

$$\frac{1}{f_1} = \frac{n-1}{R}$$

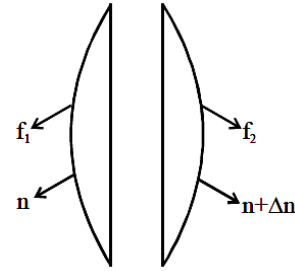
$$\frac{1}{f_2} = \frac{(n+\Delta n)-1}{R}$$

$$\frac{1}{f_{eq}} = \frac{1}{f+\Delta f} = \left(\frac{n-1}{R}\right) + \frac{(n+\Delta n)-1}{R} = \frac{2(n-1)+\Delta n}{R}$$

$$\Delta f = \left(\frac{R}{2(n-1)+\Delta n}\right) - \left(\frac{R}{2(n-1)}\right)$$

$$= \frac{R}{2} \left[ \frac{(n-1)-(n-1+\Delta n)}{(n-1+\Delta n)(n-1)} \right] = \frac{-\Delta n}{(n-1)^2} \times \frac{R}{2}$$

$$\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \quad \dots (2)$$



Relation between  $\frac{\Delta f}{f}$  and  $\frac{\Delta n}{n}$  is independent of R

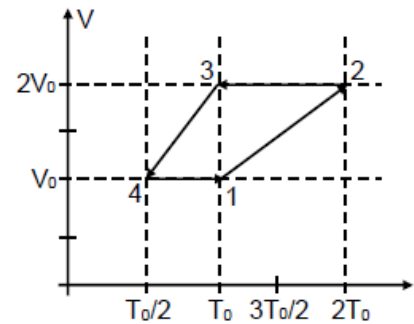
$2n-2 < n$  because  $n < 2$

$$\Rightarrow \frac{\Delta f}{f} = \frac{1}{2} \left| \frac{\Delta n}{n-1} \right| > \frac{\Delta n}{n} \text{ . So, } \frac{\Delta f}{f} > \left| \frac{\Delta n}{n} \right|$$

$$\text{Now, } |\Delta f| = \frac{f\Delta n}{(n-1)} = \frac{(20 \times 10^{-3})}{1.5-1} = 40 \times 10^{-3} = 0.04$$

If  $\frac{\Delta n}{n} < 0$ , then  $\frac{\Delta f}{f} > 0$  from equation (2).

8. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are : [R is the gas constant]



- (A) Work done in this thermodynamic cycle

$$(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1) \text{ is } |W| = \frac{1}{2} RT_0$$

- (B) The ratio of heat transfer during processes

$$1 \rightarrow 2 \text{ and } 2 \rightarrow 3 \text{ is } \left| \frac{Q_{1 \rightarrow 2}}{Q_{2 \rightarrow 3}} \right| = \frac{5}{3}$$

- (C) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

- (D) The ratio of heat transfer during processes  $1 \rightarrow 2$  and  $3 \rightarrow 4$  is  $\left| \frac{Q_{1 \rightarrow 2}}{Q_{2 \rightarrow 3}} \right| = \frac{1}{2}$

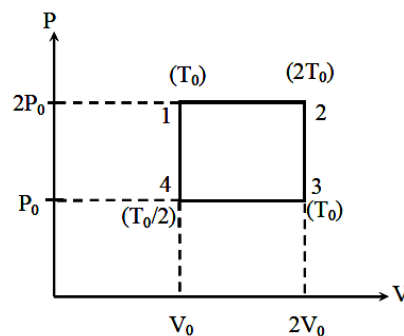
8. (A), (B)

From P-V diagram

$$W_{\text{cycle}} = P_0 V_0 = \frac{RT_0}{2}$$

$$\left| \frac{Q_{1 \rightarrow 2}}{Q_{2 \rightarrow 3}} \right| = \left| \frac{n C_P (T_2 - T_1)}{n C_V (T_3 - T_2)} \right| = \left| \frac{5}{3} \right| = \frac{5}{3}$$

$$\left| \frac{Q_{1 \rightarrow 2}}{Q_{2 \rightarrow 3}} \right| = \left| \frac{n C_P (T_2 - T_1)}{n C_V (T_4 - T_3)} \right| = 2$$



**SECTION 3 (Maximum Marks:18)**

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :  
 Full Marks : +3 If **ONLY** the correct numerical value is entered.  
 Zero Marks : 0 In all other cases.

1. A parallel plate capacitor of capacitance  $C$  has spacing  $d$  between two plates having area  $A$ . The region between the plates is filled with  $N$  dielectric layers, parallel to its plates, each with thickness  $\delta = \frac{d}{N}$ . The dielectric constant of the  $m$ th layer is  $K_m = K \left(1 + \frac{m}{N}\right)$ .

For a very large  $N (> 10^3)$ , the capacitance  $C$  is  $\alpha \left(\frac{K \epsilon_0 A}{d \ln 2}\right)$ . The value of  $\alpha$  will be \_\_\_\_\_ . [  $\epsilon_0$  is the permittivity of free space]

1. [1]

$$K_m = K \left(1 + \frac{m}{N}\right) = K \left(1 + \frac{x}{D}\right)$$

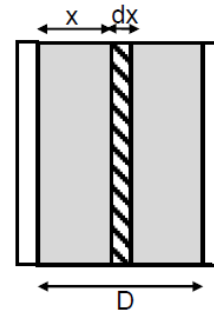
$$\frac{x}{m} = \frac{D}{N}$$

$$d\left(\frac{1}{C}\right) = \frac{dx}{K_m \epsilon_0 A} = \frac{dx}{K \epsilon_0 A \left(1 + \frac{x}{D}\right)}$$

$$\frac{1}{C_{eq}} = \int d\left(\frac{1}{C}\right) = \int_0^D \frac{D dx}{K \epsilon_0 A (D + x)}$$

$$\frac{1}{C_{eq}} = \frac{D}{K \epsilon_0 A} \ln 2$$

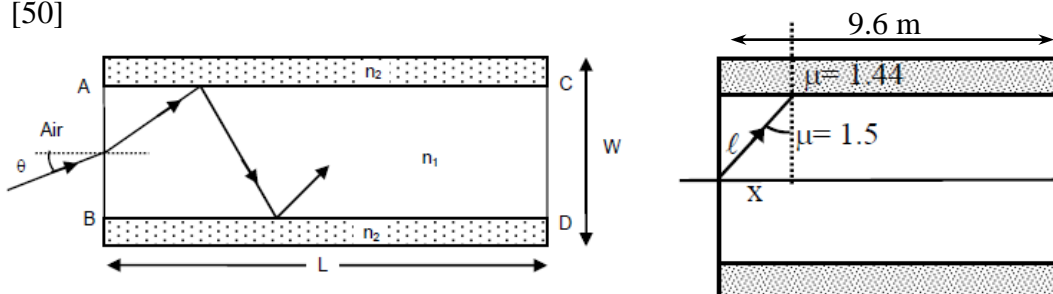
$$C_{eq} = \frac{K \epsilon_0 A}{D \ln 2}. \text{ Therefore } \alpha = 1.$$



2. A planar structure of length  $L$  and width  $W$  is made of two different optical media of refractive indices  $n_1 = 1.5$  and  $n_2 = 1.44$  as shown in figure. If  $L \gg W$ , a ray entering from end  $AB$  will emerge from end  $CD$  only if the total internal reflection condition is met inside the structure. For  $L = 9.6$  m, if the incident angle  $\theta$  is varied, the maximum time taken by a ray to exit the plane  $CD$  is  $t \times 10^{-9}$  s, where  $t$  is \_\_\_\_\_ .

[Speed of light  $c = 3 \times 10^8$  m/s]

2. [50]



$$1.5 \sin \theta_C = 1.44 \sin 90^\circ$$

$$\sin \theta_C = \frac{24}{25}$$

$$l = \frac{x}{\sin \theta_C} = \frac{25}{4}x$$

Total length for light to travel

$$l' = \frac{25}{4} \times 9.6 = 10 \text{ m}$$

$$\therefore \text{time} = \frac{l'}{C/1.5} = 5 \times 10^{-8} \text{ s} \Rightarrow 50 \times 10^{-9} \text{ s}$$

$$t = 50$$

3. A liquid at  $30^\circ \text{C}$  is poured very slowly into a Calorimeter that is at temperature of  $110^\circ \text{C}$ . The boiling temperature of the liquid is  $80^\circ \text{C}$ . It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be  $50^\circ \text{C}$ . The ratio of the Latent heat of the liquid to its specific heat will be \_\_\_\_\_  $^\circ \text{C}$ . (Neglect the heat exchange with surrounding)

3. [270]

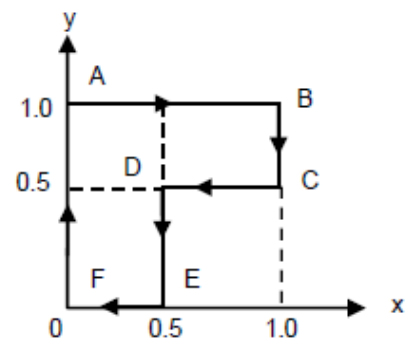
$$5(s)(50) + 5L = C(30) \quad \dots \text{ (i)}$$

$$80(s)(20) = C(30) \quad \dots \text{ (ii)}$$

$\therefore$  from (i) and (ii)

$$\frac{L}{S} = 270^\circ \text{C} \quad \therefore 270.00$$

4. A particle is moved along a path AB–BC–CD–DE–EF–FA, as shown in figure in presence of a force  $\vec{F} = (\alpha y \hat{i} + 2\alpha x \hat{j}) \text{ N}$ , where x and y are in meter and  $\alpha = -1 \text{ Nm}^{-1}$ . The work done on the particle by this force  $\vec{F}$  will be \_\_\_\_\_ joule



4. [0.75]

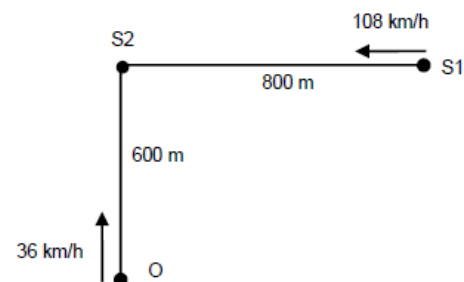
When  $\alpha = -1$

$$W_{AB} = \int_0^1 \alpha y dx = -1; \quad W_{BC} = \int_1^0.5 2\alpha x dy = +1$$

$$W_{CD} = \int_1^0.5 \alpha y dx = +0.25; \quad W_{DE} = \int_0.5^1 2\alpha x dy = +0.5$$

$$W_{EF} = W_{FA} = 0; \quad W_{\text{net}} = 0.75$$

5. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is \_\_\_\_\_ .



(Speed of the sound = 330 m/s)

5. [8.128]

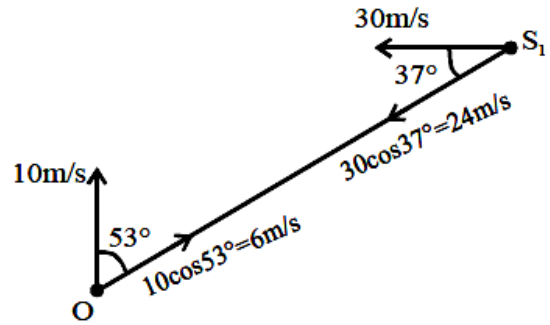
Frequency observed by O from  $S_2$

$$f_2 = \frac{330+10}{330} \times 120 = \frac{340}{330} \times 120 = 123.63 \text{ Hz}$$

frequency observed by O from  $S_1$

$$f_1 = \frac{330+6}{330-24} \times 120 = \frac{336}{306} \times 120 \approx 131.76 \text{ Hz}$$

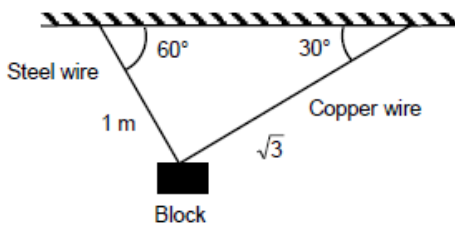
$$\text{beat frequency} = 131.76 - 123.63 = 8.128$$



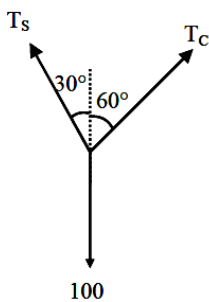
6. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area  $0.5 \text{ cm}^2$  and, length  $\sqrt{3} \text{ m}$  and  $1 \text{ m}$ , respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are  $30^\circ$  and  $60^\circ$ , respectively. If elongation in copper wire is  $(\Delta l_c)$  and elongation in

steel wire is  $(\Delta l_s)$ , then the ratio  $\frac{\Delta l_c}{\Delta l_s}$  is \_\_\_\_ .

(Young's modulus for copper and steel are  $1 \times 10^{11} \text{ N/m}^2$  and  $2 \times 10^{11} \text{ N/m}^2$ , respectively)



6. [2]



$$T_s \sin 30^\circ = T_c \sin 60^\circ$$

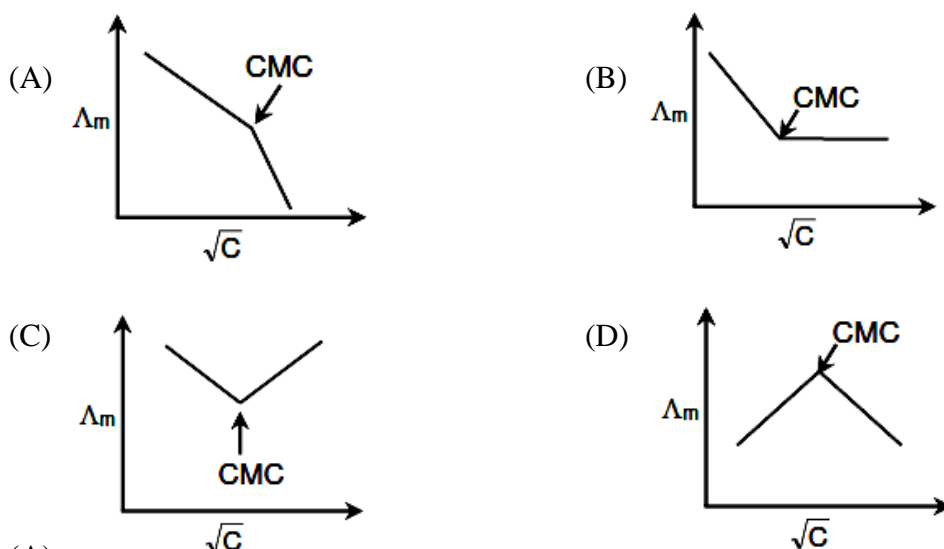
$$\frac{\Delta l_c}{\Delta l_s} = \frac{T_c l_c}{A_c Y_c} \left( \frac{A_s Y_s}{T_s l_s} \right) = 2.00 \quad [\because A_c = A_s]$$

□ □

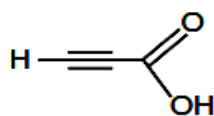
**PART II : CHEMISTRY****SECTION 1 (Maximum Marks : 12)**

- This section contains FOUR (04) questions.
- Each question has FOUR options ONLY ONE of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :  
 Full Marks : +3 If ONLY the correct option is chosen.  
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).  
 Negative Marks : -1 In all other cases.

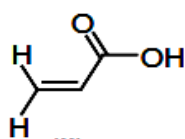
1. Molar conductivity ( $\Lambda_m$ ) of aqueous solution of sodium stearate, which behaves as a strong electrolyte is recorded at varying concentrations ( $c$ ) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution ? (critical micelle concentration (CMC) is marked with an arrow in the figures)



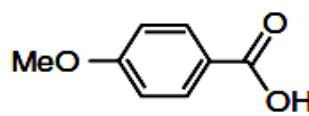
1. (A)  
 As the concentration of sodium stearate increases beyond CMC, stearate ions get clubbed together and form micelles. This abruptly causes the concentration of the current carrier anions to decrease. This is reflected by the sharp change in  $\Lambda_m$  at CMC, followed by greater rate of decrease of  $\Lambda_m$  with  $\sqrt{C}$ .
2. The green colour produced in the borax bead test of a chromium (III) salt is due to  
 (A) CrB (B) Cr<sub>2</sub>O<sub>3</sub> (C) Cr(BO<sub>2</sub>)<sub>3</sub> (D) Cr<sub>2</sub>(B<sub>4</sub>O<sub>7</sub>)<sub>3</sub>
2. (C)  
 Chromium (III) salt  $\xrightarrow{\Delta}$  Cr<sub>2</sub>O<sub>3</sub>  
 Borax  $\xrightarrow{\Delta}$  B<sub>2</sub>O<sub>3</sub> + NaBO<sub>2</sub>  
 2Cr<sub>2</sub>O<sub>3</sub> + 6B<sub>2</sub>O<sub>3</sub>  $\longrightarrow$  4Cr(BO<sub>2</sub>)<sub>3</sub>  
 So correct answer is option (C)
3. The correct order of acid strength of the following carboxylic acids is :



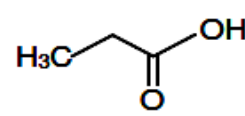
(I)



(II)



(III)



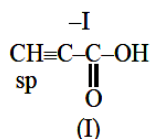
(IV)

- (A) I > II > III > IV  
 (C) I > III > II > IV

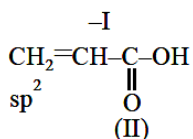
- (B) II > I > IV > III  
 (D) III > II > I > IV

3. (A)

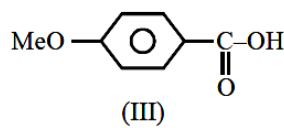
I > II > III > IV



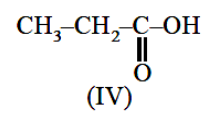
(pKa value 1.86)



(pKa value 4.3)



(pKa value 4.5)



(pKa value 4.88)

4. Calamine, malachite, magnetite and cryolite, respectively, are

- (A) ZnSO<sub>4</sub>, Cu(OH)<sub>2</sub>, Fe<sub>3</sub>O<sub>4</sub>, Na<sub>3</sub>AlF<sub>6</sub>  
 (B) ZnCO<sub>3</sub>, CuCO<sub>3</sub>, Fe<sub>2</sub>O<sub>3</sub>, Na<sub>3</sub>AlF<sub>6</sub>  
 (C) ZnSO<sub>4</sub>, CuCO<sub>3</sub>, Fe<sub>2</sub>O<sub>3</sub>, AlF<sub>3</sub>  
 (D) ZnCO<sub>3</sub>, CuCO<sub>3</sub>.Cu(OH)<sub>2</sub>, Fe<sub>3</sub>O<sub>4</sub>, Na<sub>3</sub>AlF<sub>6</sub>

4. (D)

Compound – Formula

Calamine – ZnCO<sub>3</sub>

Malachite – CuCO<sub>3</sub>.Cu(OH)<sub>2</sub>

Magnetite – Fe<sub>3</sub>O<sub>4</sub>

Cryolite – Na<sub>3</sub>AlF<sub>6</sub>

### SECTION 2 (Maximum Marks:32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the correct option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :
 

Full Marks	: +4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: +3	If all the four options are correct but <b>ONLY</b> three options are chosen.
Partial Marks	: +2	If three or more options are correct but <b>ONLY</b> two options are chosen and both of which are correct.
Partial Marks	: +1	If two or more options are correct but <b>ONLY</b> one option is chosen and it is a correct option.
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	: -1	In all other cases.

1. Each of the following options contains a set of four molecules, Identify the option(s) where all four molecules passes permanent dipole moment at room temperature.

- (A) NO<sub>2</sub>, NH<sub>3</sub>, POCl<sub>3</sub>, CH<sub>3</sub>Cl  
 (B) BF<sub>3</sub>, O<sub>3</sub>, SF<sub>6</sub>, XeF<sub>6</sub>  
 (C) BeCl<sub>2</sub>, CO<sub>2</sub>, BCl<sub>3</sub>, CHCl<sub>3</sub>  
 (D) SO<sub>2</sub>, C<sub>6</sub>H<sub>5</sub>Cl, H<sub>2</sub>Se, BrF<sub>5</sub>

1. (A), (D)

Polar Molecule

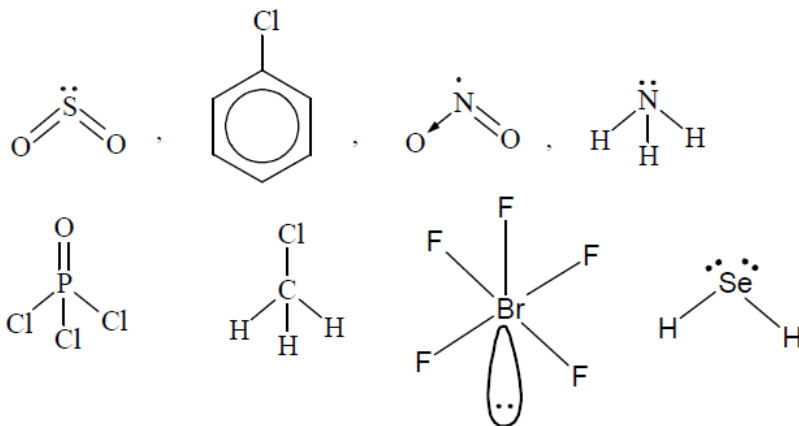
Non-polar molecule

CHCl<sub>3</sub>, SO<sub>2</sub>, C<sub>6</sub>H<sub>5</sub>Cl,

BeCl<sub>2</sub>, CO<sub>2</sub>, BCl<sub>3</sub>, SF<sub>6</sub>

H<sub>2</sub>Se, BrF<sub>5</sub>, O<sub>3</sub>, XeF<sub>6</sub>,

NO<sub>2</sub>, NH<sub>3</sub>, POCl<sub>3</sub>, CH<sub>3</sub>Cl



2. Which of the following statement(s) is(are) correct regarding the root mean square speed ( $U_{\text{rms}}$ ) and average translational kinetic energy ( $\epsilon_{\text{av}}$ ) of a molecule in a gas at equilibrium
- (A)  $U_{\text{rms}}$  is inversely proportional to the square root of its molecular mass  
 (B)  $\epsilon_{\text{av}}$  is doubled when its temperature is increased four times.  
 (C)  $U_{\text{rms}}$  is doubled when its temperature is increased four times.  
 (D)  $\epsilon_{\text{av}}$  at a given temperature does not depend on its molecular mass.
2. (A), (C), (D)

$$\epsilon_{\text{av}} = \frac{3}{2}RT \quad U_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \text{and} \quad U_{\text{rms}} \propto \frac{1}{\sqrt{M}}$$

$\therefore \epsilon_{\text{av}}$  doesn't depend on its molecular mass

3. Which of the following statements(s) is(are) true ?
- (A) The two six-membered cyclic hemiacetal forms of D-(+)-glucose are called anomers.  
 (B) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose.  
 (C) Monosaccharides cannot be hydrolysed to give polyhydroxy aldehydes and ketones.  
 (D) Oxidation of glucose with bromine water gives glutamic acid.
3. (A), (B), (C)

(A) True

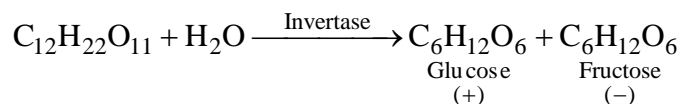


$\alpha$ -D-glucopyranose

$\beta$ -D-glucopyranose

$\alpha$ -D-glucopyranose and  $\beta$ -D-glucopyranose are anomers of each other.

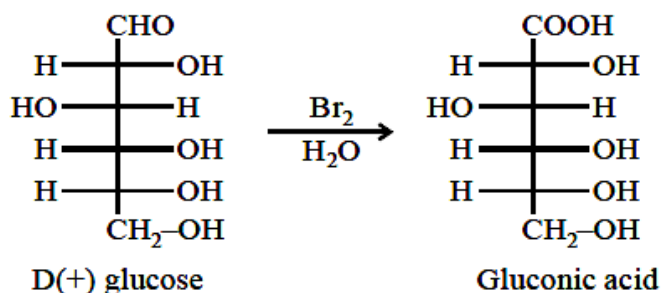
(B) True



(C) True

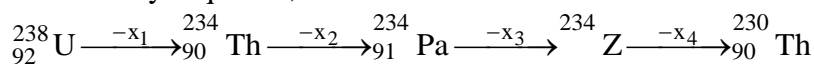
Monosaccharide cannot be hydrolysed to give polyhydroxy aldehydes and ketones.

(D) False



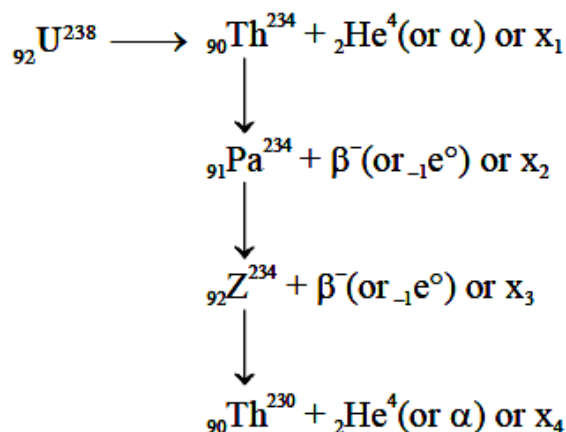


4. In the decay sequence,



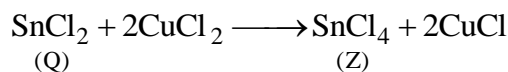
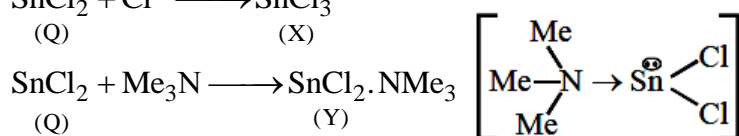
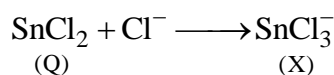
$x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  are particles/radiation emitted by the respective isotopes. The correct option(s) is(are):

- (A)  $x_1$  will deflect towards negatively charged plate.  
 (B)  $x_2$  is  $\beta^-$   
 (C)  $x_3$  is  $\gamma$ -ray  
 (D) z is an isotope of uranium
4. (A), (B), (D)



5. A tin chloride Q undergoes the following reaction (not balanced)  $\text{Q} + \text{Cl}^- \rightarrow \text{X}$  is monoanion having pyramidal geometry. Both Y and Z are neutral compounds. Choose the correct option(s)
- (A) The central atom in X is  $\text{sp}^3$  hybridized.  
 (B) There is a coordinate bond in Y  
 (C) The oxidation state of the central atom in Z is +2  
 (D) The central atom in Z has one lone pair of electrons.

5. (A), (B)



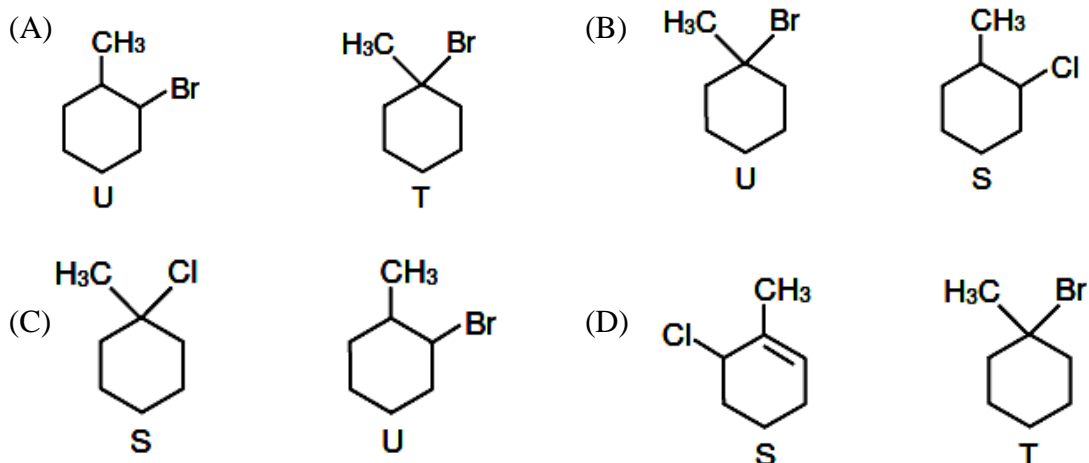
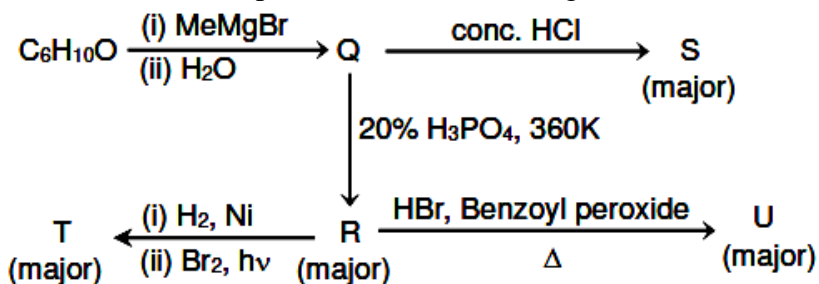
6. Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation

- (A)  $\frac{1}{8}\text{S}_{8(s)} + \text{O}_2(g) \rightarrow \text{SO}_2(g)$   (B)  $2\text{H}_2(g) + \text{O}_2(g) \rightarrow 2\text{H}_2\text{O}(l)$   
 (C)  $\frac{3}{2}\text{O}_2(g) \rightarrow \text{O}_3(g)$   (D)  $2\text{C}(g) + 3\text{H}_2(g) \rightarrow \text{C}_2\text{H}_6(g)$

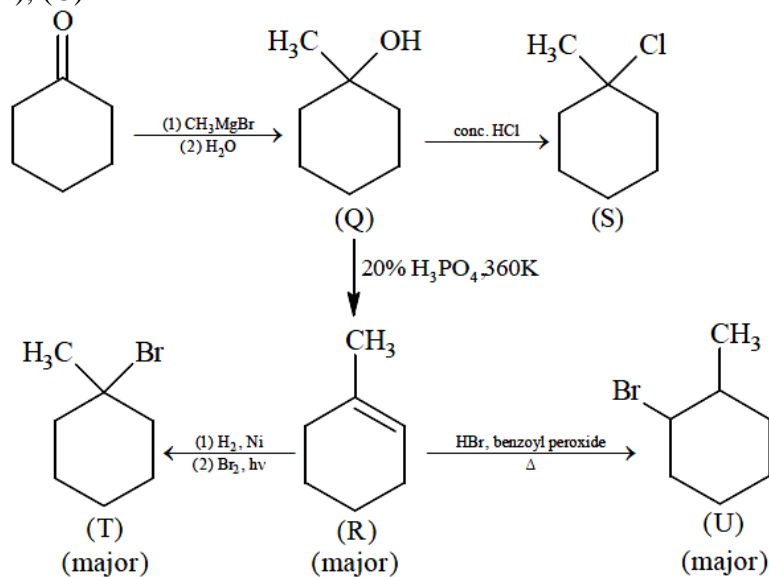
6. (A), (C)

Standard enthalpy of formation of a compound is the standard enthalpy when one mole of a compound is formed from the elements in their stable state.

7. Choose the correct option(s) for the following set of reactions



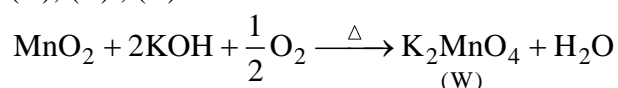
7. (A), (C)

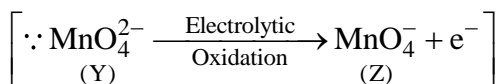
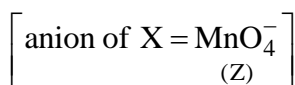
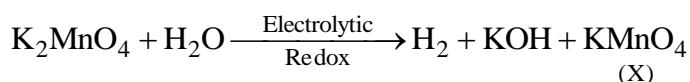
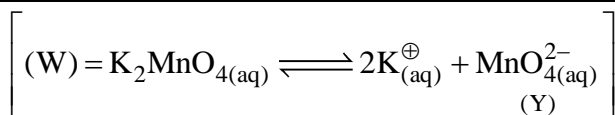


8. Fusion of  $\text{MnO}_2$  with  $\text{KOH}$  in presence of  $\text{O}_2$  produces a salt  $\text{W}$ . Alkaline solution of  $\text{W}$  upon electrolytic oxidation yields another salt  $\text{X}$ . The manganese containing ions present in  $\text{W}$  and  $\text{X}$ , respectively are  $\text{Y}$  and  $\text{Z}$ . Correct statement(s) is (are)

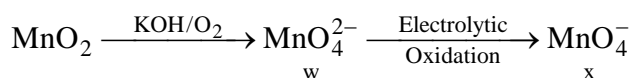
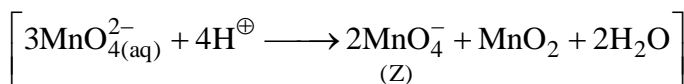
- (A) In both  $\text{Y}$  and  $\text{Z}$ ,  $\pi$ -bonding occurs between  $p$ -orbitals of oxygen and  $d$ -orbitals of manganese  
 (B) Both  $\text{Y}$  and  $\text{Z}$  are coloured and have tetrahedral shape  
 (C) In aqueous acidic solution,  $\text{Y}$  undergoes disproportionation reaction to give  $\text{Z}$  and  $\text{MnO}_2$   
 (D)  $\text{Y}$  is diamagnetic in nature while  $\text{Z}$  is paramagnetic

8. (A), (B), (C)

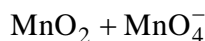




$\therefore$  In acidic solution ; Y undergoes disproportionation reaction



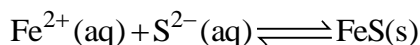
acidic medium



### SECTION 3 (Maximum Marks:18)

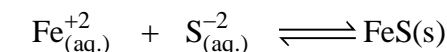
- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :  
 Full Marks : +3 If **ONLY** the correct numerical value is entered.  
 Zero Marks : 0 In all other cases.

1. For the following reaction, equilibrium constant  $K_c$  at 298 K is  $1.6 \times 10^{17}$



When equal volume of 0.06 M  $\text{Fe}^{2+}(\text{aq})$  and 0.2 M  $\text{S}^{2-}(\text{aq})$  solution are mixed, then equilibrium concentration of  $\text{Fe}^{2+}(\text{aq})$  is found to be  $Y \times 10^{-17}$  M. The value of Y is \_\_\_\_\_

1. [8.92 or 8.93]



$$0.03 \text{ M} \quad 0.1 \text{ M}$$

$$(0.03 - x) \quad (0.1 - x)$$

$$\approx y \quad \approx 0.07$$

$$K_c \gg 10^3 \Rightarrow 0.03 - x \approx 0 \approx y \Rightarrow x = 0.03$$

$$K_c = 1.6 \times 10^{17} = \frac{1}{y \times 0.07}$$

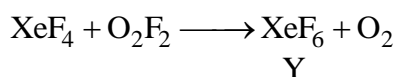
$$y = \frac{10^{-17}}{1.6 \times 0.07} = 8.928 \times 10^{-17} = Y \times 10^{-17}$$

$$y \approx 8.93$$

**(20) Vidyalankar : IIT JEE 2019 – Advanced : Question Paper & Solution**

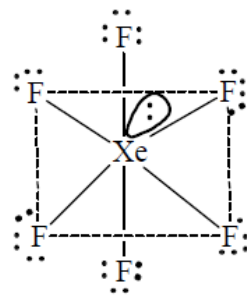
2. At 143 K, the reaction of  $\text{XeF}_4$  with  $\text{O}_2\text{F}_2$  produces a xenon compound Y. The total number of lone pair(s) electrons present on the whole molecule of Y is \_\_\_\_\_ .

2. [19]

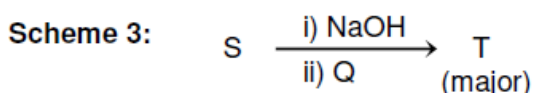
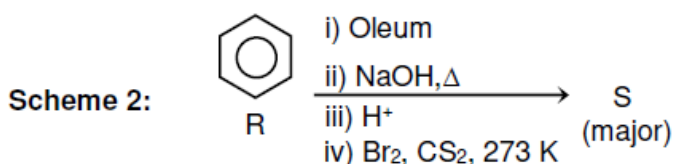
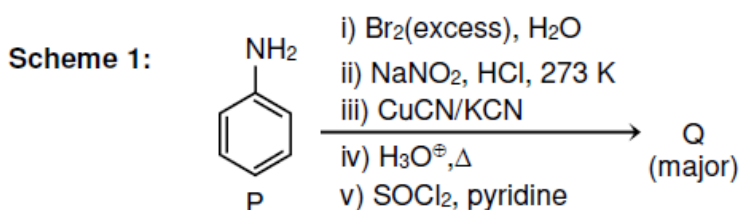


Y has 3 lone pair of electron in each fluorine and one lone pair of electron in xenon.

Hence total lone pair of electrons is 19.

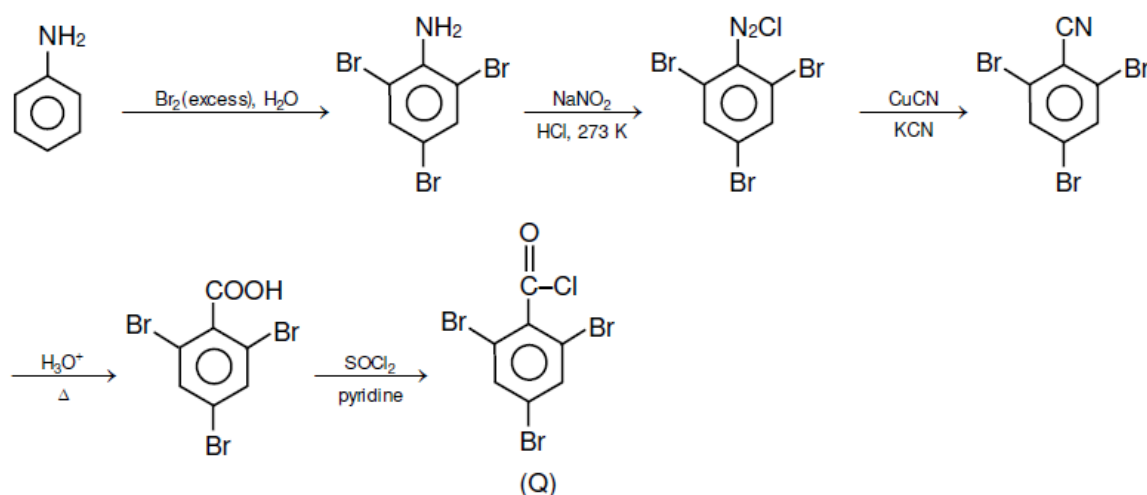


3. Schemes 1 and 2 describe the conversion of P to Q and R to S, respectively. Scheme 3 describes the synthesis of T from Q and S. The total number of Br atoms in a molecule of T is \_\_\_\_\_ .

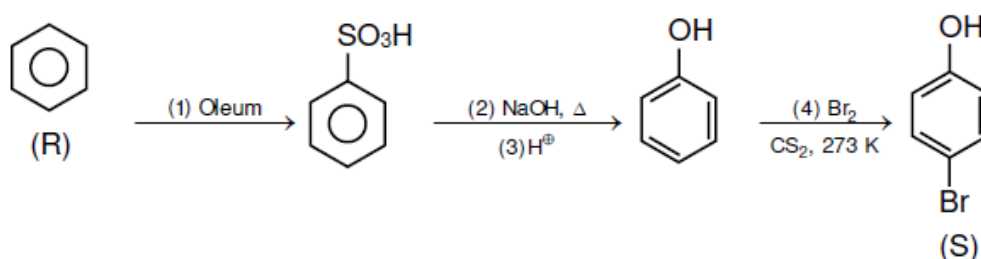


3. [4]

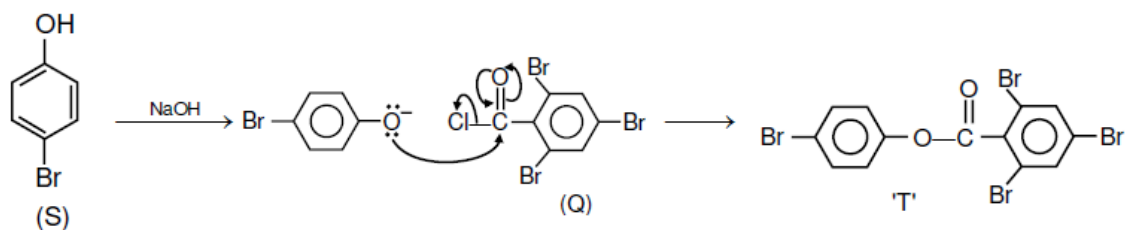
Scheme 1 :



Scheme 2 :



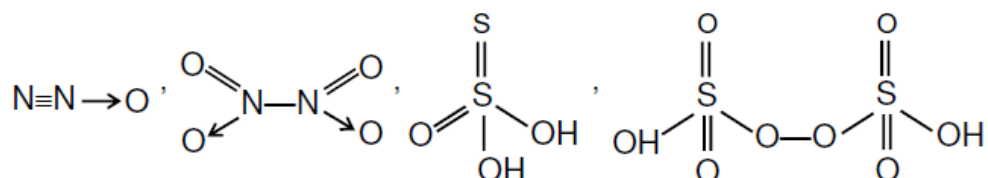
Scheme 3 :



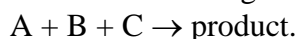
4. Among  $B_2H_6$ ,  $B_3N_3H_6$ ,  $N_2O$ ,  $N_2O_4$ ,  $H_2S_2O_3$  and  $H_2S_2O_8$ , the total number of molecules containing covalent bond between two atoms of the same kind is \_\_\_\_\_

4. [4]

$N_2O_4$ ,  $H_2S_2O_3$ ,  $N_2O$ ,  $H_2S_2O_8$



5. Consider the kinetic data given in the following table for the reaction :



Experiment No.	[A] ( $\text{mol dm}^{-3}$ )	[B] ( $\text{mol dm}^{-3}$ )	[C] ( $\text{mol dm}^{-3}$ )	Rate of reaction ( $\text{mol dm}^{-3} \text{s}^{-1}$ )
1	0.2	0.1	0.1	$6.0 \times 10^{-5}$
2	0.2	0.2	0.1	$6.0 \times 10^{-5}$
3	0.2	0.1	0.2	$1.2 \times 10^{-4}$
4	0.3	0.1	0.1	$9.0 \times 10^{-5}$

The rate of the reaction for  $[A] = 0.15 \text{ mol dm}^{-3}$ ,  $[B] = 0.25 \text{ mol dm}^{-3}$  and  $[C] = 0.15 \text{ mol dm}^{-3}$  is found to be  $Y \times 10^{-5} \text{ mol dm}^{-3} \text{s}^{-1}$ . The value of Y is \_\_\_\_\_

5. [6.75]

$$\text{Rate} = k[A]^x [B]^y [C]^z$$

By exp. No. 1 and 2  $y = 0$

By exp. No. 1 and 3  $z = 1$

By exp. No. 1 and 4  $x = 1$

$$\text{Rate} = k[A]^1 [B]^0 [C]^1$$

From Exp. No. 1 :  $6 \times 10^{-5} = k(0.2)(0.1)$

$$\Rightarrow k = 3 \times 10^{-3}$$

Now for  $[A] = 0.15$   $[B] = 0.25$   $[C] = 0.15$

$$\text{Rate} = k[A]^1 [B]^0 [C]^1 = 3 \times 10^{-3} \times 0.15 \times 1 \times 0.15$$

6. On dissolving 0.5 g of a non-volatile non-ionic solute to 39 g of benzene, its vapor pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of benzene (in K) upon addition of the solute is \_\_\_\_\_.

(Given data : Molar mass and the molal freezing point depression constant of benzene are  $78 \text{ g mol}^{-1}$  and  $5.12 \text{ K kg mol}^{-1}$ , respectively)

6. [1.02 or 1.03]

$$\frac{P^0 - P_s}{P^0} = \frac{n_{\text{solute}}}{n_{\text{solute}} + n_{\text{solvent}}} ; \quad \frac{650 - 640}{650} = \frac{n_{\text{solute}}}{n_{\text{solute}} + 0.5}$$

$$n_{\text{solute}} = \left( \frac{5}{640} \right) ; \quad \text{Molality} = \frac{5 \times 1000}{640 \times 39}$$

$$\Delta T_f = m \times K_b = \frac{5.12 \times 5 \times 1000}{640 \times 39} = 1.0256$$

$$\Delta T_f \approx 1.03$$

**PART III – MATHEMATICS****SECTION 1 (Maximum Marks:12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :  
 Full Marks : +3 If **ONLY** the correct option is chosen.  
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).  
 Negative Marks : -1 In all other cases.

1. A line  $y = mx + 1$  intersect the circle  $(x - 3)^2 + (y + 2)^2 = 25$  at points P and Q. If the midpoint of the line segment PQ has x-coordinate  $-\frac{3}{5}$ , then which one of the following options is correct.

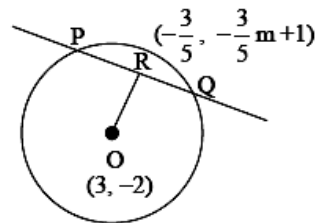
- (A)  $6 \leq m < 8$                       (B)  $2 \leq m < 4$                       (C)  $4 \leq m < 6$                       (D)  $-3 \leq m < -1$

1. (B)

$$PQ \perp OR \Rightarrow \text{Slope } OR = -\frac{1}{m} = \frac{-\frac{3}{5}m + 1 + 2}{-\frac{3}{5} - 3}$$

$$\Rightarrow m^2 - 5m + 6 = 0$$

$$\Rightarrow m = 2, 3$$



2. Let  $M = \begin{bmatrix} \sin^4 \theta & -1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where  $\alpha = \alpha(\theta)$  and  $\beta = \beta(\theta)$  are real numbers and I is the  $2 \times 2$  identity matrix.

If  $\alpha^*$  is the minimum of the set  $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$  and

$\beta^*$  is the minimum of the set  $\{\beta(\theta) : \theta \in [0, 2\pi)\}$

Then the value of  $\alpha^* + \beta^*$  is

- (A)  $\frac{-37}{16}$                       (B)  $\frac{-29}{16}$                       (C)  $\frac{-31}{16}$                       (D)  $\frac{-17}{16}$

2. (B)

$$M = \alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta \begin{bmatrix} \cos^4 \theta & 1 + \sin^2 \theta \\ -1 - \cos^2 \theta & \sin^4 \theta \end{bmatrix}$$

On comparing we have

$$\sin^4 \theta = \alpha + \frac{\beta \cos^4 \theta}{|M|}$$

$$-1 - \sin^2 \theta = \alpha + \frac{\beta(1 + \sin^2 \theta)}{|M|}$$

$$\alpha = \sin^4 \theta + \cos^4 \theta$$

$$\text{Now } \alpha = (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$= 1 - \frac{\sin^2 2\theta}{2}$$

$$\Rightarrow \alpha^* = \frac{1}{2}$$

We have,  $|M| = \sin^4 \theta \cos^4 \theta + (1 + \sin^2 \theta)(1 + \cos^2 \theta)$   
 $= 2 + \sin^2 \theta \cos^2 \theta + \sin^4 \theta \cos^4 \theta$   
 $= (\sin^2 \theta \cos^2 \theta + 1/2)^2 + 7/4$

$\Rightarrow \beta = -|M| = -\frac{7}{4} - \left(\sin^2 \theta \cos^2 \theta + \frac{1}{2}\right)^2$

$\Rightarrow \beta^* = -\frac{7}{4} - \left(\frac{1}{4} + \frac{1}{2}\right)^2 = -\frac{7}{4} - \frac{9}{16} = -\frac{37}{16}$

$\alpha^* + \beta^* = -\frac{29}{16}$ .

3. Let S be the set of all complex numbers z satisfying  $|z-2+i| \geq \sqrt{5}$ . If the complex number  $z_0$  is such that  $\frac{1}{|z_0-1|}$  is the maximum of the set  $\left\{\frac{1}{|z-1|} : z \in S\right\}$ , then the

principal argument of  $\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}$  is

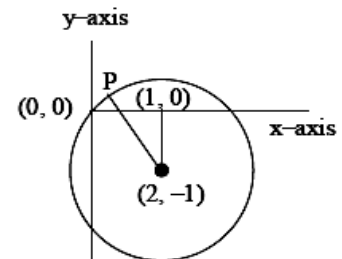
- (A)  $\frac{\pi}{4}$                       (B)  $\frac{3\pi}{4}$                       (C)  $-\frac{\pi}{2}$                       (D)  $\frac{\pi}{2}$

3. (C)

Clearly location of required point  $z_0$  is at P with abscissa  $< 1$  and ordinate  $> 0$

Now  $\arg \left[ \frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i} \right] = \text{Arg} \left( \frac{x-2}{y+1} \right) i = \text{Arg} ki$  and  $k < 0$

$\Rightarrow$  Required argument  $= -\pi/2$

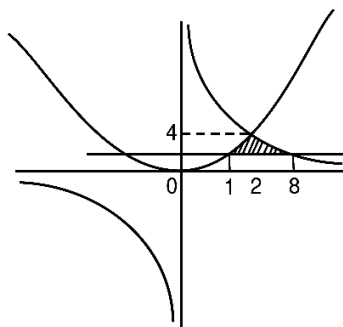


4. The area of the region  $\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$  is

- (A)  $16 \log_e 2 - 6$                       (B)  $8 \log_e 2 - \frac{7}{3}$                       (C)  $16 \log_e 2 - \frac{14}{3}$                       (D)  $8 \log_e 2 - \frac{14}{3}$

4. (C)

$xy \leq 8$   
 $1 \leq y \leq x^2$   
 $x^2 \cdot x = 8$   
 $x = 2$



Required Area  $= \int_1^4 \left( \frac{8}{y} - \sqrt{y} \right) dy = \left[ 8 \ln y - \frac{y^{3/2}}{3/2} \right]_1^4 = 8 \ln 4 - \frac{2}{3} \cdot 8 - 0 + \frac{2}{3} = 16 \ln 2 - \frac{14}{3}$

**SECTION 2 (Maximum Marks:32)**

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the correct option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :  
Full Marks : +4 If only (all) the correct option(s) is (are) chosen.  
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen.  
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen and both of which are correct.  
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option.  
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).  
Negative Marks : -1 In all other cases.

1. Define the collections  $\{E_1, E_2, E_3, \dots\}$  of ellipses and  $\{R_1, R_2, R_3, \dots\}$  of rectangles of follows :

$$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1;$$

$R_1$  : Rectangle of largest area with sides parallel to the axes, inscribed in  $E_1$  :

$$E_n : \text{ellipse } \frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1 \text{ of largest area inscribed in } R_{n-1}, n > 1$$

$R_n$  : rectangle of largest area, with sides parallel to the axes, inscribed in  $E_n, n > 1$

Then which of the following options is/are correct ?

(A) The eccentricities of  $E_{18}$  and  $E_{19}$  are NOT equal

(B) The length of latus rectum of  $E_9$  is  $\frac{1}{6}$

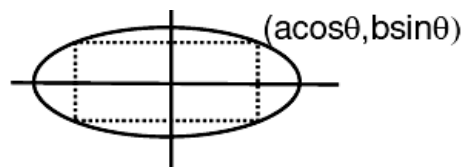
(C)  $\sum_{n=1}^N (\text{area of } R_n) < 24$ , for each positive integer N

(D) The distance of a focus from the centre in  $E_9$  is  $\frac{\sqrt{5}}{32}$

1. (B), (C)

Area Maximum when  $\theta = 45^\circ$

	a	b
$E_1$	3	2
$E_2$	$\frac{3}{\sqrt{2}}$	$\frac{2}{\sqrt{2}}$
$E_3$	$\frac{3}{(\sqrt{2})^2}$	$\frac{2}{(\sqrt{2})^2}$
$E_9$	$\frac{3}{(\sqrt{2})^8}$	$\frac{2}{(\sqrt{2})^8}$





(A)  $e_{18} = e_{19}$

(B) Length of LR is ellipse =  $\frac{2b^2}{a} = 2 \cdot \frac{4 \cdot 2^4}{2^8 \cdot 3} = \frac{1}{6}$

(C)  $R_1 + R_2 + \dots + R_N < R_1 + R_2 + \dots + R_N + \dots = \frac{2ab}{1 - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}} = 4ab = 4 \cdot 3 \cdot 2 = 24$

(D) Distance between focus and center of ellipse =  $a_9 e_9 = \frac{3}{2^4} \cdot \frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{16}$

2. In a non-right-angled triangle  $\Delta PQR$ , Let  $p, q, r$  denote the lengths of the sides opposite to the angles at  $P, Q, R$  respectively. The median from  $R$  meets the side  $PQ$  at  $S$ , the perpendicular from  $P$  meets the side  $QR$  at  $E$ , and  $RS$  and  $PE$  intersect at  $O$ . If  $p = \sqrt{3}, q = 1$ , and the radius of the circumcircle of the  $\Delta PQR$  equals 1, then which of the following options is/are correct ?

(A) Length of  $RS = \frac{\sqrt{7}}{2}$  (B) Area of  $\Delta SOE = \frac{\sqrt{3}}{12}$

(C) Radius of incircle of  $\Delta PQR = \frac{\sqrt{3}}{2}(2 - \sqrt{3})$

(D) Length of  $OE = \frac{1}{6}$

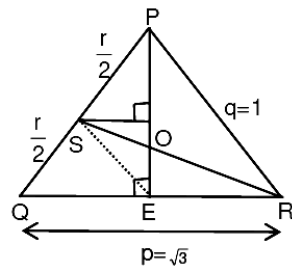
2. (A), (C), (D)

$\frac{p}{\sin P} = \frac{q}{\sin Q} = 2(1) \Rightarrow \sin P = \frac{\sqrt{3}}{2}, \sin Q = \frac{1}{2}$

$\Rightarrow \angle P = 60^\circ$  or  $120^\circ$  and  $\angle Q = 30^\circ$  or  $150^\circ$

because  $\angle P + \angle Q$  must be less than  $180^\circ$  but not equal to  $90^\circ$

$\angle P = 120^\circ$  and  $\angle Q = 30^\circ$  and  $\angle R = 30^\circ$   $\frac{r}{\sin R} = 2 \Rightarrow r = 1$



Now length of median  $RS = \frac{1}{2}\sqrt{2p^2 + 2q^2 - r^2} = \frac{1}{2}\sqrt{6 + 2 - 1} = \frac{\sqrt{7}}{2} \Rightarrow$  option (A) is correct

In radius =  $\frac{2\Delta}{p+q+r} = \frac{2pqr}{4 \times (1)} = \frac{1}{2} \left( \frac{1 \times 1 \times \sqrt{3}}{1+1+\sqrt{3}} \right) = \frac{\sqrt{3}}{2} \left( \frac{2 - \sqrt{3}}{1} \right) \Rightarrow$  option (C) is correct

$\Rightarrow \frac{1}{2} \times \sqrt{3} \times PE = \frac{pqr}{4(1)}$  (equal area of  $\Delta$ )  $\Rightarrow PE = \frac{1 \times 1 \times \sqrt{3}}{4} \times \frac{2}{\sqrt{3}} = \frac{1}{2}$

$\Rightarrow OE = \frac{2(\text{Area of } \Delta OQR)}{QR} = \frac{2 \times \frac{1}{3} \left( \frac{1}{2} \cdot 1 \cdot \sqrt{3} \sin 30^\circ \right)}{\sqrt{3}} = \frac{1}{6}$

3. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1 & 0 \leq x < 1 \\ (2/3)x^3 - 4x^2 + 7x - (8/3) & 1 \leq x < 3 \\ (x - 2)\ln(x - 2) - x + (10/3) & x \geq 3 \end{cases}$$

Then which of the following options is/are correct?

(A)  $f'$  is NOT differentiable at  $x = 1$

(B)  $f$  is increasing on  $(-\infty, 0)$

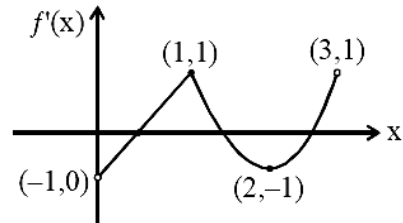
(C)  $f$  is onto

(D)  $f'$  has a local maximum at  $x = 1$

3. (A), (C), (D)

$$f(x) = \begin{cases} (x+1)^5 - 2x, & x < 0; \\ x^2 - x + 1, & 0 \leq x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \leq x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \geq 3 \end{cases}$$

$$f'(x) = \begin{cases} 5(x+1)^4 - 2 & x < 0 \\ 2x - 1 & 0 \leq x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3} & 1 \leq x < 3 \\ (x-2)\ln(x-2) - x + \frac{10}{3} & x \geq 3 \end{cases}$$



$x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1$  takes value between  $-\infty$  to 1

Also  $(x-2)\ln(x-2) - x + \frac{10}{3}$  takes value between  $\frac{1}{3}$  to  $\infty$

So, range of  $f(x)$  is  $\mathbb{R}$ .

$f'(1^-) = 2$  and  $f'(1^+) = -4$ . So  $f'(x)$  is non-diff at  $x = 1$

$f'(x)$  has local maxima at  $x = 1 \leq$

4. Let  $\alpha$  and  $\beta$  be the roots of  $x^2 - x - 1 = 0$  with  $\alpha > \beta$ . For all positive integers  $n$ , define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1 \quad \text{and} \quad b_1 = 1 \quad \text{and} \quad b_n = a_n = a_{n-1} + a_{n+1}, n \geq 2$$

the which of the following options is/are correct?

(A)  $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$  (B)  $b_n = \alpha^n + \beta^n$  for all  $n \geq 1$

(C)  $a_1 + a_2 + \dots + a_n = a_{n+2} - 1$  for all  $n \geq 1$  (D)  $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

4. (A), (B), (C)

$$(A) b_n = a_{n+1} + a_{n-1} = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} + \frac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} = \frac{\alpha^{n-1}(\alpha^2 + 1) - \beta^{n-1}(\beta^2 + 1)}{\alpha - \beta}$$

$$= \frac{\alpha^{n-1}(\alpha + 2) - \beta^{n-1}(\beta + 2)}{\alpha - \beta} = \frac{\alpha^{n-1} \left( \frac{5 + \sqrt{5}}{2} \right) - \beta^{n-1} \left( \frac{5 - \sqrt{5}}{2} \right)}{\alpha - \beta}$$

$$= \frac{\sqrt{5}\alpha^{n-1} \left( \frac{\sqrt{5} + 1}{2} \right) - \sqrt{5}\beta^{n-1} \left( \frac{\sqrt{5} - 1}{2} \right)}{\alpha - \beta} = \frac{\sqrt{5}(\alpha^n + \beta^n)}{\alpha - \beta} = \alpha^n + \beta^n \quad \because \alpha - \beta = \sqrt{5}$$

$$(B) \sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \left( \frac{\alpha}{10} \right)^n + \sum_{n=1}^{\infty} \left( \frac{\beta}{10} \right)^n = \frac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \frac{\alpha}{10 - \alpha} + \frac{\beta}{10 - \beta}$$

$$= \frac{10(\alpha + \beta) - 2\alpha\beta}{100 - 10(\alpha + \beta) + \alpha\beta} = \frac{10 + 2}{89} = \frac{12}{89}$$

$$(C) \sum_{n=1}^{\infty} \frac{a_n}{10^n} = \sum \frac{\alpha^n - \beta^n}{(\alpha - \beta)10^n} = \frac{1}{\alpha - \beta} \left( \frac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} \right) \frac{1}{\alpha - \beta} \left( \frac{\alpha}{10 - \alpha} - \frac{\beta}{10 - \beta} \right)$$

$$= \frac{1}{\alpha - \beta} \cdot \frac{(10(\alpha - \beta) - \alpha\beta + \alpha\beta)}{100 - 10(\alpha + \beta) + \alpha\beta} = \frac{10}{89} \quad \text{Option (C) is correct.}$$

$$(D) a_1 + a_2 + \dots + a_n = \sum a_i = \frac{\sum \alpha^i - \sum \beta^i}{\alpha - \beta} = \frac{\frac{\alpha(1 - \alpha^n)}{1 - \alpha} - \frac{\beta(1 - \beta^n)}{1 - \beta}}{\alpha - \beta}$$

$$= \frac{(\alpha + 1)(1 - \alpha^n) - (\beta + 1)(1 - \beta^n)}{(1 - \alpha)(1 - \beta)(\alpha - \beta)} = \frac{\alpha^2 - \alpha^{n+2} - \beta^2 + \beta^{n+2}}{(1 - \alpha)(1 - \beta)(\alpha - \beta)} = \frac{\sqrt{5} + \beta^{n+2} - \alpha^{n+2}}{\beta - \alpha} = -1 + a_{n+2}$$

5. Let  $L_1$  and  $L_2$  denote the lines  $\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k})$ ,  $\lambda \in \mathbb{R}$  and  $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k})$ ,  $\mu \in \mathbb{R}$  respectively. If  $L_3$  is a line which is perpendicular to both  $L_1$  and  $L_2$  and cuts both of them, then which of the following options describe (s)  $L_3$ ?

(A)  $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$ ,  $t \in \mathbb{R}$       (B)  $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$ ,  $t \in \mathbb{R}$

(C)  $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$ ,  $t \in \mathbb{R}$       (D)  $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k})$ ,  $t \in \mathbb{R}$

5. (A), (B), (C)

Both given lines are skew lines.

So direction ratios of any line perpendicular to these lines are  $6\hat{i} + 6\hat{j} - 3\hat{k}$

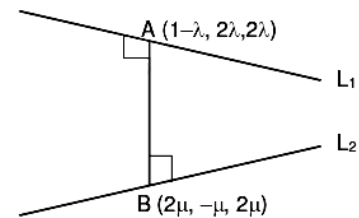
i.e 2, 2, -1

Points at shortest distance between given lines are

$\overline{AB} \perp$  line  $L_1$

$\overline{AB} \perp$  line  $L_2$

So  $A\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$



Now equation of required line  $\vec{r} = \left(\frac{8}{9}\hat{i} + \frac{2}{9}\hat{j} + \frac{2}{9}\hat{k}\right) + \alpha(2\hat{i} + 2\hat{j} - \hat{k})$

6. There are three bags  $B_1$ ,  $B_2$  and  $B_3$ . The bag  $B_1$  contains 5 red and 5 green balls.  $B_2$  contains 3 red and 5 green balls and  $B_3$  contains 5 red and 3 green balls. Bags  $B_1$ ,  $B_2$  and  $B_3$  have probabilities  $3/10$ ,  $3/10$  and  $4/10$  respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

(A) Probability that the chosen ball is green, given that the selected bag is  $B_3$ , equals  $\frac{3}{8}$

(B) Probability that the selected bag is  $B_3$ , given that the chosen ball is green, equals  $\frac{5}{13}$

(C) Probability that the chosen ball is green equals  $\frac{39}{80}$

(D) Probability that the selected bag is  $B_3$ , given that the chosen ball is green, equals  $\frac{3}{10}$

6. (A), (C)

	Bag – 1	Bag – 2	Bag – 3
Red Balls	5	3	5
Green Balls	5	5	3
Total	10	8	8

$$(A) P(\text{Ball is Green}) = P(B_1) P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)$$

$$= \frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8} = \frac{39}{80}$$

$$(B) P(\text{Ball chosen is Green/ Ball is from 3}^{\text{rd}} \text{ Bag}) = \frac{3}{8}$$

(C), (D)

$P(\text{Ball is from 3}^{\text{rd}} \text{ Bag/ Ball chosen is Green})$

$$= \frac{P(B_3)P(G/B_3)}{P(B_1)P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)}$$

$$P(B_1) = \frac{3}{10}$$

$$P(B_2) = \frac{3}{10}$$

$$P(B_3) = \frac{4}{10} = \frac{\frac{4}{10} \times \frac{3}{8}}{\frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8}} = \frac{4}{13}$$

7. Let  $\Gamma$  denote a curve  $y = f(x)$  which is in the first quadrant and let the point  $(1, 0)$  lie on it. Let the tangent to  $\Gamma$  at a point  $P$  intersect the  $y$ -axis at  $Y_P$ . If  $PY_P$  has length 1 for each point  $P$  on  $\Gamma$ . Then which of the following options is/are correct?

$$(A) y = -\ln\left(\frac{1 + \sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$$

$$(B) xy' + \sqrt{1-x^2} = 0$$

$$(C) xy' - \sqrt{1-x^2} = 0$$

$$(D) y = \ln\left(\frac{1 + \sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$$

7. (B), (D)

Equation tangent

$$y - y_1 = \frac{dy_1}{dx_1} (x - x_1)$$

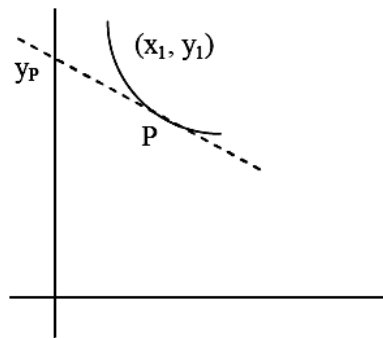
$$y_p \left( 0, y_1 - x_1 \frac{dy_1}{dx_1} \right)$$

$$Py_p = \sqrt{x_1^2 + \left(-x_1 \frac{dy_1}{dx_1}\right)^2} = 1$$

$$\Rightarrow \frac{dy}{dx} = \pm \frac{\sqrt{1-x^2}}{x} \Rightarrow dy = \pm \int \frac{\sqrt{1-x^2}}{x} dx, \quad 1-x^2 = t^2, \quad -x dx = t dt$$

$$dy = \pm \int \frac{-t^2}{1-t^2} dt$$

$$y = \pm \left( \int 1 dt + \int \frac{1}{1-t^2} dt \right)$$



$$y = \pm \left( t - \frac{1}{2} \ln \left| \frac{1+t}{1-t} \right| \right) + c$$

$$y = \pm \left( \sqrt{1-x^2} - \frac{1}{2} \ln \left| \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \right| \right) + c$$

y is passing through (1, 0) so c = 0

$$y = \pm \left( \sqrt{1-x^2} - \frac{1}{2} \ln \left( \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} \right) \right)$$

$$y = \pm \left( \sqrt{1-x^2} - \ln \left( \frac{1+\sqrt{1-x^2}}{x} \right) \right)$$

8. Let  $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$  and  $\text{adj} M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$  where a and b are real numbers. Which

of the following options is/are correct?

(A)  $\det(\text{adj} M^2) = 81$

(B)  $a + b = 3$

(C) If  $M \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ , then  $\alpha - \beta + \gamma = 3$

(D)  $(\text{adj} M)^{-1} + \text{adj} M^{-1} = -M$

8. (B), (C), (D)

$$(\text{adj} M)_{11} = 2 - 3b = -1 \Rightarrow b = 1$$

$$\text{Also, } (\text{adj} M)_{22} = -3a = -6 \Rightarrow a = 2$$

$$\text{Now, } \det M = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -2$$

$$\Rightarrow \det(\text{adj} M^2) = (\det M^2)^2$$

$$= (\det M)^4 = 16$$

$$\text{Also } M^{-1} = \frac{\text{adj} M}{\det M}$$

$$\Rightarrow \text{adj} M = -2M^{-1}$$

$$\Rightarrow (\text{adj} M)^{-1} = \frac{-1}{2} M$$

$$\text{And, } \text{adj}(M^{-1}) = (M^{-1})^{-1} \det(M^{-1}) = \frac{1}{\det M} M = \frac{-M}{2}$$

$$\text{Hence, } (\text{adj} M)^{-1} + \text{adj}(M^{-1}) = -M$$

$$\text{Further, } MX = b$$

$$\Rightarrow X = M^{-1}b = \frac{-\text{adj} M}{2} b$$

$$= \frac{-1}{2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{-1}{2} \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow (\alpha, \beta, \gamma) = (1, -1, 1)$$

**SECTION 3 (Maximum Marks:18)**

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :  
Full Marks : +3 If **ONLY** the correct numerical value is entered.  
Zero Marks : **0** In all other cases.

1. Three lines are given by

$$\vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R}$$

$$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in \mathbb{R} \text{ and } \vec{r} = \nu(\hat{i} + \hat{j} + \hat{k}), \nu \in \mathbb{R}$$

Let the lines cut the plane  $x + y + z = 1$  at the points A, B and C respectively. If the area of the triangle ABC is  $\Delta$  then the value of  $(6\Delta)^2$  equals \_\_\_\_\_.

1. [0.75]

Put  $(\lambda, 0, 0)$  in  $x + y + z = 1 \Rightarrow \lambda = 1 \Rightarrow P(1, 0, 0)$

Put  $(\mu, \mu, 0) \Rightarrow 2\mu = 1 \Rightarrow Q\left(\frac{1}{2}, \frac{1}{2}, 0\right)$

Put  $(\gamma, \gamma, \gamma) \Rightarrow \gamma = \frac{1}{3} \Rightarrow R\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$$\text{Area of triangle PQR} = \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2} \left| \left( \frac{\hat{i} - \hat{j}}{2} \right) \times \left( \frac{2\hat{i} - \hat{j} - \hat{k}}{3} \right) \right| = \frac{1}{12} |\hat{i} + \hat{j} + \hat{k}| = \frac{\sqrt{3}}{12}$$

$$\Rightarrow (6\Delta)^2 = 0.75$$

2. Let S be the sample space of all  $3 \times 3$  matrices with entries from the set  $\{0, 1\}$ . Let the events

$$E_1 = \{A \in S : \det A = 0\} \text{ and}$$

$$E_2 = \{A \in S : \text{Sum of entries of } A \text{ is } 7\}$$

If a matrix is chosen at random from S, then the conditional probability  $P(E_1 | E_2)$  equals \_\_\_\_\_.

2. [0.50]

$$n(E_2) = \text{arrangement of } 7, 1 \text{ and } 2 \quad \text{or} \quad = \frac{9!}{7!2!} = 36$$

$n(E_1 \cap E_2)$  = both zero should be in a row or a column

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ (number of ways of arranging of } (1, 0, 0) = 3 \text{ and arrangement of row} = 3$$

$$\text{Total} = 9$$

In same way for  $(1, 0, 0)$  for columns number of ways will be = 9

$$\text{Total ways} = 18$$

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{18}{36} = \frac{1}{2} = 0.50$$

3. That  $\omega \neq 1$  be a cube root of unity. Then the minimum of the set  $\{|a + b\omega + c\omega^2|^2; a, b, c \text{ are distinct non zero integers}\}$  equals \_\_\_\_\_.

3. [3]

$$\begin{aligned} |a + b\omega + c\omega^2|^2 &= (a + b\omega + c\omega^2)^2 \overline{(a + b\omega + c\omega^2)} \\ &= (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega) \\ &= a^2 + b^2 + c^2 - ab - bc - ca \\ &= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] \\ &\geq \frac{1+1+4}{2} = 3 \quad (\text{when } a = 1, b = 2, c = 3) \end{aligned}$$

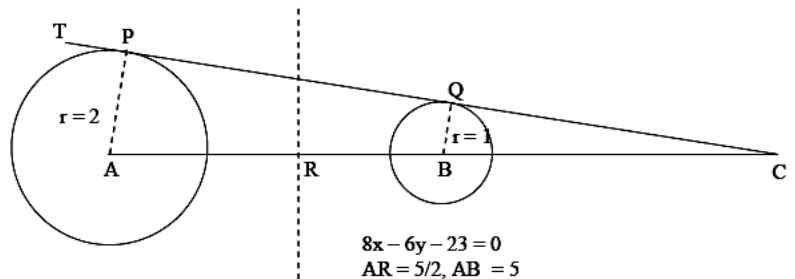
4. Let the point B be the reflection of the point A (2, 3) with respect to the line  $8x - 6y - 23 = 0$ . Let  $\Gamma_A$  and  $\Gamma_B$  be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles  $\Gamma_A$  and  $\Gamma_B$  such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is \_\_\_\_\_

4. [10]

Now  $\Delta APC$  and  $BQC$  are similarly

$$\frac{BC}{AC} = \frac{1}{2} \Rightarrow 2(AC - AB) = AC$$

$$AC = 2AB = 10$$



5. Let AP (a, d) denote the set of all the term of an infinite arithmetic progression with first term a and common difference  $d > 0$ . If  $AP(1, 3) \cap AP(2, 5) \cap AP(3, 7) = AP(a, d)$  then  $a + d$  equals \_\_\_\_\_.

5. [157]

$$AP(1, 3) \equiv \{1, 4, 7, 10, \dots\} = \{n/n = 3k + 1, k \in \mathbb{W}\}$$

$$AP(2, 5) \equiv \{2, 7, 12, \dots\} = \{n/n = 3k + 1, k \in \mathbb{W}\}$$

$$AP(3, 7) \equiv \{3, 10, 17, \dots\} = \{n/n = 7k + 3, k \in \mathbb{W}\}$$

Let common term is M

$$M \equiv 1 \pmod{3}, M \equiv 2 \pmod{5}, M \equiv 3 \pmod{7}$$

$$\Rightarrow M \equiv 52 \pmod{105}$$

$$\text{So, } a = 52, d = 105 \text{ and } a + d = 157$$

6.  $I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)}$  then find  $27I^2$  equals \_\_\_\_\_.

6. [4]

$$I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)} \quad \dots(1)$$

By a + b - x property

$$I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)} = \frac{2}{\pi} \int_{\pi/4}^{-\pi/4} \frac{e^{\sin x} dx}{(1 + e^{\sin x})(2 - \cos 2x)} \quad \dots(2)$$

Adding (1) and (2)

$$2I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{(1 + e^{\sin x})}{(1 + e^{\sin x})(2 - \cos 2x)} dx \Rightarrow I = \frac{1}{\pi} \int_{\pi/4}^{-\pi/4} \frac{1}{2 - (2\cos^2 x - 1)} dx = \frac{1}{\pi} \int_{\pi/4}^{-\pi/4} \frac{\sec^2 x}{3\sec^2 x - 2} dx$$

Put,  $\tan x = t$ ,  $\sec^2 x dx = dt$

$$= \frac{2}{\pi} \int_0^1 \frac{dt}{3t^2 + 1} = \frac{2}{3\pi} \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} \left( \tan^{-1} \left( \frac{t}{1/\sqrt{3}} \right) \right)_0^1 = \frac{2}{\sqrt{3}\pi} \left( \tan^{-1}(\sqrt{3}) - \tan^{-1}(0) \right) = \frac{2}{\sqrt{3}\pi} \left( \frac{\pi}{3} \right) = \frac{2}{3\sqrt{3}}$$

$$\text{Now, } 27I^2 = 27 \times \frac{4}{27} = 4$$

□ □ □ □ □ □