Vidyalankar

F.E. Sem. I

Applied Mathematics - I

List of Formulae

1. De-Moivre′s Theorem

The value or one of th value of $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

2. Binomial Theorem

$(a + b)^n = nc_0\,(a)^n\,(b)^0 + nc_1(a)^{n-1}\,(b) + nc_2(a)^{n-2}\,(b)^2 + \ldots + nc_n(a)^n(b)^0$

where $nc_0 = nc_n = 1$

$nc_1 = nc_{n-1} = n$

$nc_2 = nc_{n-2} = \dfrac{n(n-1)}{2!}$

$nc_3 = nc_{n-3} = \dfrac{n(n-1)(n-2)}{3!}\ldots$

3. $z = x + iy$ [cartesian form]

$= r(\cos\theta + i\sin\theta)$ [Polar Form]

$= r\,e^{i\theta}$ [Expontial form]

where $r = \sqrt{x^2 + y^2}$

and $\theta = \alpha$ if z lies in I^st^ Quadrant

$= \pi - \alpha$ if z lies in II^nd^ Quadrant

$= \pi + \alpha$ if z lies in III^rd^ Quadrant

$= 2\pi - \alpha$ if z lies in IV^th^ Quadrant, where $\alpha = \tan^{-1}\left|\dfrac{y}{x}\right|$

4. $\sinh x = \dfrac{e^x - e^{-x}}{2}$

5. $\text{cash } x = \dfrac{e^x + e^{-x}}{2}$

6. $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

7. $\coth x = \dfrac{1}{\text{yanh}\,x}$

8. $\text{sech } x = \dfrac{1}{\cosh x}$

9. $\text{cosech } x = \dfrac{1}{\sinh x}$

10. $\sin(ix) = i\sinh x$

11. $\cos(ix) = \cosh x$

12. $\tan(ix) = i\tanh x$

13. $\cot(ix) = -i\coth x$

14. $\sec(ix) = \text{sech } x$

15. $\text{cosec}(ix) = -\,i\text{cosech } x$

16. $\sinh(ix) = i \sin x$

17. $\cosh(ix) = \cos x$

18. $\tanh(ix) = i \tan x$

19. $\coth(ix) = -i \cot x$

20. $\text{sech}(ix) = \sec x$

21. $\text{cosech}(ix) = -i \,\text{cosec}\, x$

22. $\sinh^{-1} z = \log\left(z + \sqrt{z^2 + 1}\right)$

23. $\cosh^{-1} z = \log\left(z + \sqrt{z^2 - 1}\right)$

24. $\tanh^{-1} z = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$

25. $\coth^{-1} z = \tanh^{-1}\left(\frac{1}{z}\right)$

26. $\text{sech}^{-1} z = \cosh^{-1}\left(\frac{1}{z}\right)$

27. $\text{cosech}^{-1} z = \sinh^{-1}\left(\frac{1}{z}\right)$

28. $\cosh^2 x - \sinh^2 x = 1$

29. $1 - \tanh^2 x = \text{sech}^2 x$

30. $1 - \coth^2 x = -\text{cosech}^2 x$

31. $\sinh 2x = 2 \sinh x \cosh x = \frac{2\tanh x}{1-\tanh^2 x}$

32. $\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x = \frac{1+\tanh^2 x}{1-\tanh^2 x}$

33. $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$

34. $\sinh 3x = 3\sinh x + 4\sinh^3 x$

35. $\cosh 3x = 4\cosh^3 x - 3\cosh x$

36. $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

37. $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

38. $\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

39. $\sinh C + \sinh D = 2\sinh\left(\frac{C+D}{2}\right)\cosh\left(\frac{C-D}{2}\right)$

40. $\sinh C - \sinh D = 2\cosh\left(\frac{C+D}{2}\right)\sinh\left(\frac{C-D}{2}\right)$

41. $\cosh C + \cosh D = 2\cosh\left(\frac{C+D}{2}\right)\cosh\left(\frac{C-D}{2}\right)$

42. $\cosh C - \cosh D = 2\sinh\left(\frac{C+D}{2}\right)\sinh\left(\frac{C-D}{2}\right)$

43. $2\sinh A \cosh B = \sinh(A+B) + \sinh(A-B)$

44. $2\cosh A \sinh B = \sinh(A+B) - \sinh(A-B)$

45. $2 \cosh A \cosh B = \cosh(A + B) + \cosh(A - B)$
46. $2 \sinh A \sinh B = \cosh(A + B) - \cosh(A - B)$
47. $\sinh(x + 2\pi i) = \sinh x$
48. $\sinh(x + 2n\pi i) = \sinh x$
49. $\cosh(x + 2\pi i) = \cosh x$
50. $\cosh(x + 2n\pi i) = \cosh x$
51. $\tanh(x + \pi i) = \tanh x$
52. $\tanh(x + n\pi i) = \tanh x$

Derivative :

	y	$\frac{dy}{dx}$
53.	sinh x	+ cosh x
54.	cosh x	+ sinh x
55.	tanh x	$+ \text{sech}^2 x$
56.	coth x	$- \text{cosech}^2 x$
57.	sechx	– sech x tanh x
58.	cosech x	– cosech x coth x

59. $\log(x + iy) = \frac{1}{2}\log\left(x^2 + y^2\right) + i\tan^{-1}\left(\frac{y}{x}\right)$, x, y > 0 (Principle value)

60. $\log(x + iy) = \frac{1}{2}\log(x^2 + y^2) + i\left[2n\pi + \tan^{-1}\left(\frac{y}{x}\right)\right]$, x, y > 0 (General value)

Matrices
61. symmetric matrix : $A' = A$
62. skew symmetric matrix : $A' = -A$
63. Hermitian Matrix : $A^\theta = A$, where $A^\theta = (\bar{A})' = \overline{(A')}$
64. skew Hermitian Matrix : $A^\theta = -A$
65. Orthogonal Matrix : $AA' = A'A = I$
66. Unitary Matrix : $AA^\theta = A^\theta A = I$
67. If A is orthogonal then $A^{-1} = A'$
68. If A is unitary then $A^{-1} = A^\theta$
69. System of Non-Homogeneous equations
If $\rho(A) < \rho(A : B) \Rightarrow$ system is inconsistent and has No solution
If $\rho(A) = \rho(A : B) \Rightarrow$ system is consistent
(i) If $n = r \Rightarrow$ system has Unique solution.
(ii) If $n < r \Rightarrow$ system has infinitely many solutions.
70. System of homogeneous equations.
If $\rho(A) = n \Rightarrow$ system has ZERO solution
If $\rho(A) < n \Rightarrow$ system has Non zero solution

Successive Differentiation
71. If $y = (ax + b)^m$
Then $y_n = (m)(m - 1)(m - 2)(m - 3) \ldots (m - (n - 1))(ax + b)^{m-n}(a)^m$, m > n
$= m!\,(a)^m$ if m = n
$= 0$ if m < n

72. If $y = x^m$

Then $y_n = (m)(m-1)(m-2)(m-3)\ldots(m-(n-1))\,x^{m-n}$, $m > n$

$= m!$ if $m = n$

$= 0$ if $m < n$

73. If $y = a^{mx}$, Then $y_n = (m \log a)^n a^{mx}$

74. If $y = e^{mx}$, Then $y_n = (m)^n e^{mx}$

75. If $y = \sin(ax + b)$ Then, $y_n = (a)^n \sin\left(ax + b + \frac{n\pi}{2}\right)$

76. If $y = \cos(ax + b)$ Then, $y_n = (a)^n \cos\left(ax + b + \frac{n\pi}{2}\right)$

77. If $y = e^{ax} \sin(bx + c)$ Then

$y_n = r^n e^{ax} \sin(bx + c + n\phi)$, where $r = \sqrt{a^2 + b^2}$, $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

78. If $y = e^{ax} \cos(bx + c)$ Then

$y_n = r^n e^{ax} \cos(bx + c + n\phi)$, where $r = \sqrt{a^2 + b^2}$, $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

79. If $y = \frac{1}{(ax+b)}$, Then $y_n = \frac{(-1)^n n!(a)^n}{(ax+b)^{n+1}}$

80. If $y = \log(ax + b)$, Then $y_n = \frac{(-1)^{n-1}(n-1)!(a)^n}{(ax+b)^n}$

81. If $y = \frac{1}{(ax+b)^m}$, Then $y_n = \frac{(-1)^n (m+n-1)!(a)^n}{(m-1)!(ax+b)^{m+n}}$

82. Leibnit′z Theorem

If u and v are n^{th} differentiable functions of x

Then n^{th} derivative of their product (uv) is given by

$(uv)_n = u_n v + nc_1 u_{n-1} v_1 + nc_2 u_{n-2} v_2 + nc_3 u_{n-3} v_3 + \ldots + uv_n$

Partial Derivatives

83. If $f(x, y) = 0$, Then $\frac{dy}{dx} = \frac{-f_x}{f_y}$

84. If u is Homogeneous Function of degree n in x and y, Then,

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)\,u$

85. If f(u) is Homogeneous function of degree n in x and y, Then

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = G(u)\,[G'u - 1]$, where $G(u) = \frac{nf(u)}{f'(u)}$

86. If u is Homogeneous function of degree n in x, y, z, Then

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z} = nu$

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}+z^2\frac{\partial^2 u}{\partial z^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+2yz\frac{\partial^2 u}{\partial y\partial z}+2zx\frac{\partial^2 u}{\partial z\partial x} = n(n-1)\,u$

87. If f(u) is homogeneous function of degree n in x, y, z. Then

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z} = \frac{nf(u)}{f'(u)}$

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}+z^2\frac{\partial^2 u}{\partial z^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+2yz\frac{\partial^2 u}{\partial y\partial z}+2zx\frac{\partial^2 u}{\partial z\partial x} = G(u)\,[G'(u)-1]$

where $G(u) = \frac{nf(u)}{f'(u)}$

88. (i) If $u = f_1(x, y)$, $v = f_2(x, y)$ Then,

$$J = J\left(\frac{u,v}{x,y}\right) = \partial\left(\frac{u,v}{x,y}\right) = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

(ii) If $x = g_1(u, v)$, $y = g_2(u, v)$ Then,

$$J^* = J\left(\frac{x,y}{u,v}\right) = \partial\left(\frac{x,y}{u,v}\right) = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

and $JJ^* = 1$

89. If $\phi_1 = f_1(u, v, x) = 0$

$\phi_2 = f_2(u, v, y) = 0$

then, $\frac{\partial u}{\partial x} = -\frac{J\left(\frac{\phi_1,\phi_2}{x,v}\right)}{J\left(\frac{\phi_1,\phi_2}{u,v}\right)}$, $\frac{\partial u}{\partial y} = -\frac{J\left(\frac{\phi_1,\phi_2}{y,v}\right)}{J\left(\frac{\phi_1,\phi_2}{u,v}\right)}$

90. If $\phi_1 = f_1(u, v, w, x) = 0$

$\phi_2 = f_2(u, v, w, y) = 0$

$\phi_3 = f_3(u, v, w, z) = 0$

Then $\frac{\partial u}{\partial x} = -\frac{J\left(\frac{\phi_1,\phi_2,\phi_3}{x,v,w}\right)}{J\left(\frac{\phi_1,\phi_2,\phi_3}{u,v,w}\right)}$, $\frac{\partial u}{\partial y} = -\frac{J\left(\frac{\phi_1,\phi_2,\phi_3}{y,v,w}\right)}{J\left(\frac{\phi_1,\phi_2,\phi_3}{u,v,w}\right)}$, $\frac{\partial u}{\partial z} = -\frac{J\left(\frac{\phi_1,\phi_2,\phi_3}{z,v,w}\right)}{J\left(\frac{\phi_1,\phi_2,\phi_3}{u,v,w}\right)}$

91. If $u \to y \to x$ Then $\frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx}$

92. If $\phi \to u, v \to x$ Then $\frac{d\phi}{dx} = \frac{\partial\phi}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial\phi}{\partial v}\frac{dv}{dx}$

93. If $\phi \to u \to x, y$ Then $\frac{\partial\phi}{\partial x} = \frac{d\phi}{du}\frac{\partial u}{\partial x}$

$\frac{\partial\phi}{\partial y} = \frac{d\phi}{du}\frac{\partial u}{\partial y}$

94. If $\phi \to u, v \to x, y$

Then
$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial v}\frac{\partial v}{\partial x}$$
$$\frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial \phi}{\partial v}\frac{\partial v}{\partial y}$$

95. If $\phi \to u, v, w \to x, y, z$

Then
$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial \phi}{\partial w}\frac{\partial w}{\partial x}$$
$$\frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial \phi}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial \phi}{\partial w}\frac{\partial w}{\partial y}$$
$$\frac{\partial \phi}{\partial z} = \frac{\partial \phi}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial \phi}{\partial v}\frac{\partial v}{\partial z} + \frac{\partial \phi}{\partial w}\frac{\partial w}{\partial z}$$

96. $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

97. $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} - \ldots$

98. $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots$

99. $\log(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \ldots$

100. $(1 + x)^n = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \ldots$

101. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

102. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

103. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots$

104. $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$

105. $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots$

106. $\tanh x = x - \frac{x^3}{3} + \frac{2}{15}x^5 - \ldots$

107. $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$

108. $\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots$

109. $\sin^{-1}x = x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+...$

110. $\sinh^{-1}x = x-\frac{x^3}{6}+\frac{3}{40}x^5-\frac{5}{112}x^7+...$

111. $\cos^{-1}x = \frac{\pi}{2}-\left[x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+...\right]$

112. $\cosh^{-1}x = \frac{\pi i}{2}-\left[x-\frac{x^3}{6}+\frac{3}{40}x^5-\frac{5}{112}x^7+...\right]$

113. Maclaurin's Series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{\partial^3}{3!}f'''(0) + ...$$

114. Taylor's Series

$$f(x + h) = f(h)+xf'(h)+\frac{x^2}{2!}f''(h)+\frac{x^3}{3!}f'''(h)+...$$

$$f(x + h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+...$$

$$f(x) = f(h) + (x-h) f'(h) + \frac{(x-h)^2}{2!}f''(h)+\frac{(x-h)^3}{3!}f'''(h)+...$$

115. $\lim_{x\to 0}\frac{\sin x}{x} = 1$

116. $\lim_{x\to 0}\frac{\sin^{-1}x}{x} = 1$

117. $\lim_{x\to 0}\frac{\tan x}{x} = 1$

118. $\lim_{x\to 0}\frac{\tan^{-1}x}{x} = 1$

119. $\lim_{x\to 0}\frac{\sinh x}{x} = 1$

120. $\lim_{x\to 0}\frac{\sinh^{-1}x}{x} = 1$

121. $\lim_{x\to 0}\frac{\tanh x}{x} = 1$

122. $\lim_{x\to 0}\frac{\tanh^{-1}x}{x} = 1$

123. $\lim_{x\to 0}(1+x)^{1/x} = e$

124. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

Fitting of curves

125. Straight Line : $y = a + bx$

Normal Equations : $\Sigma y = aN + b\Sigma x$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

126. Parabola : $y = a + bx + cx^2$

Normal equations : $\Sigma y = aN + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

127. Exponential curve

(i) $y = ae^{bx}$

$\log y = \log a + bx$

$Y = A + B X$ where $Y = \log y, X = x$

$A = \log a, B = b$

Normal Equations : $\Sigma Y = AN + B\Sigma X$

$\Sigma XY = A\Sigma x + B\Sigma x^2$

(ii) $y = ab^x$

$\log y = \log a + x \log b$

$Y = A + Bx$

Normal Equations : $\Sigma Y = AN + B\Sigma X$

$\Sigma xY = A\Sigma x + B\Sigma x^2$

(iii) $y = ax^b$

$\log y = \log a + b \log x$

$Y = A + Bx$

Normal Equations : $\Sigma Y = AN + B\Sigma x$

$\Sigma xY = A\Sigma x + B\Sigma x^2$

Trigonometry

128. In a circle of a radius r, an arc subtends an angle θ (in radians) at the centre of the circle, then

(i) length of arc = s $= r\theta$

(ii) area of the sector = A $= \frac{1}{2}r^2\theta$

129. Fundamental Identities :

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$ (iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

130.

	$0° = 0^c$	$30° = \frac{\pi^c}{6}$	$45° = \frac{\pi^c}{4}$	$60° = \frac{\pi^c}{3}$	$90° = \frac{\pi^c}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

131. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

132. $\sin C + \sin D = 2\sin\left[\frac{C+D}{2}\right].\cos\left[\frac{C-D}{2}\right]$

$\sin C - \sin D = 2\cos\left[\frac{C+D}{2}\right].\sin\left[\frac{C-D}{2}\right]$

$\cos C + \cos D = 2\cos\left[\frac{C+D}{2}\right].\cos\left[\frac{C-D}{2}\right]$

$\cos C - \cos D = -2\sin\left[\frac{C+D}{2}\right].\sin\left[\frac{C-D}{2}\right]$

133. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
$2\cos A\sin B = \sin(A+B) - \sin(A-B)$
$2\cos A\cos B = \cos(A+B) + \cos(A-B)$
$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

134. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\sin^2 A = \frac{1-\cos 2A}{2}$

$\cos^2 A = \frac{1+\cos 2A}{2}$

135. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$; $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$; $\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$; $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

136. $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ $a > 0, b > 0, ab < 1.$

$\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a-b}{1+ab}\right)$, $a > 0, b > 0.$

137. (i) $\sin^{-1}(\sin x) = x$ (ii) $\sin(\sin^{-1} x) = x$
(iii) $\cos^{-1}(\cos x) = x$ (iv) $\cos(\cos^{-1} x) = x$ etc.

138. (i) $\sin^{-1}(\cos x) = \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$

(ii) $\cos^{-1}(\sin x) = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$

139. (i) $\sin^{-1}(-x) = -\sin^{-1}x$ (ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$

140. (i) $\operatorname{cosec}^{-1}\left(\frac{a}{b}\right) = \sin^{-1}\left(\frac{b}{a}\right)$ (ii) $\sec^{-1}\left(\frac{a}{b}\right) = \cos^{-1}\left(\frac{b}{a}\right)$

(iii) $\cot^{-1}\left(\frac{a}{b}\right) = \tan^{-1}\left(\frac{b}{a}\right)$

141. (i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ (ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(iii) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

Calculus
Derivatives of the following functions w.r.t. x.

y or f(x)	$\frac{dy}{dx}$ or f ′(x)	y or f(x)	$\frac{dy}{dx}$ or f ′(x)
c (const.)	0	sec x	sec x . tan x
x^n	$n \,.\, x^{n-1}$	cot x	$-\operatorname{cosec}^2 x$
$(ax+b)^n$	$n(ax+b)^{n-1} \,.\, a$	$\sin^{-1}x$	$\frac{1}{\sqrt{(1-x^2)}}$
$u \pm v$	$\frac{du}{dx} \pm \frac{dv}{dx}$	$\cos^{-1}x$	$\frac{-1}{\sqrt{(1-x^2)}}$
u . v	$u\frac{dv}{dx} + v\frac{du}{dx}$	$\tan^{-1}x$	$\frac{1}{1+x^2}$
cu	$c\frac{du}{dx}$	$\operatorname{cosec}^{-1}x$	$\frac{-1}{\lvert x\sqrt{(x^2-1)}\rvert}$, $\lvert x \rvert > 1$
u . v . w	$vw\frac{du}{dx} + wu\frac{dv}{dx} + uv\frac{dw}{dx}$	$\sec^{-1}x$	$\frac{1}{\lvert x\sqrt{(x^2-1)}\rvert}$, $\lvert x \rvert > 1$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\cot^{-1}x$	$\frac{-1}{1+x^2}$
sin x	cos x	e^x	e^x
cos x	– sin x	a^x	$a^x \log a$
tan x	$\sec^2 x$	$\log_e x$	$\frac{1}{x}$
cosec x	– cosec x . cot x	$\log_a x$	$\frac{1}{x \log_e a}$

❑ ❑ ❑ ❑ ❑